

# THETA PRODUCTS AND ETA QUOTIENTS OF LEVEL 24 AND WEIGHT 2

AYŞE ALACA, ŞABAN ALACA, ZAFER SELCUK AYGIN

**ABSTRACT.** We find bases for the spaces  $M_2\left(\Gamma_0(24), \left(\frac{d}{\cdot}\right)\right)$  ( $d = 1, 8, 12, 24$ ) of modular forms. We determine the Fourier coefficients of all 35 theta products  $\varphi[a_1, a_2, a_3, a_4](z)$  in these spaces. We then deduce formulas for the number of representations of a positive integer  $n$  by diagonal quaternary quadratic forms with coefficients 1, 2, 3 or 6 in a uniform manner, of which 14 are Ramanujan's universal quaternary quadratic forms. We also find all the eta quotients in the Eisenstein spaces  $E_2\left(\Gamma_0(24), \left(\frac{d}{\cdot}\right)\right)$  ( $d = 1, 8, 12, 24$ ) and give their Fourier coefficients.

**Keywords and phrases:** Dedekind eta function, eta quotients, theta products, Eisenstein series, modular forms, cusp forms, Fourier coefficients, Fourier series.

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## 1. INTRODUCTION AND NOTATION

Let  $\mathbb{N}$ ,  $\mathbb{N}_0$ ,  $\mathbb{Z}$ ,  $\mathbb{Q}$  and  $\mathbb{C}$  denote the sets of positive integers, non-negative integers, integers, rational numbers and complex numbers, respectively. Let  $N \in \mathbb{N}$ . Let  $\Gamma_0(N)$  be the modular subgroup defined by

$$\Gamma_0(N) = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \mid a, b, c, d \in \mathbb{Z}, \ ad - bc = 1, \ c \equiv 0 \pmod{N} \right\}.$$

We define a Dirichlet character  $\chi_t$  for each  $t \in \{-24, -8, -4, -3, 1, 8, 12, 24\}$  by

$$(1.1) \quad \chi_t(m) = \left(\frac{t}{m}\right) \text{ for } m \in \mathbb{Z}.$$

Note that  $\chi_1$  is the trivial character. Let  $\chi$  and  $\psi$  be Dirichlet characters. For  $n \in \mathbb{N}$  we define the generalized sums of divisors functions  $\sigma_{(\chi, \psi)}(n)$  by

$$(1.2) \quad \sigma_{(\chi, \psi)}(n) := \sum_{1 \leq m|n} \chi(m) \psi(n/m) m.$$

If  $n \notin \mathbb{N}$  we set  $\sigma_{(\chi, \psi)}(n) = 0$ . If  $\chi = \psi = \chi_1$ , then  $\sigma_{(\chi, \psi)}(n)$  coincides with the sum of divisors function

$$\sigma(n) = \sum_{1 \leq m|n} m.$$

Let  $k \in \mathbb{Z}$ . We write  $M_k(\Gamma_0(N), \chi)$  to denote the space of modular forms of weight  $k$  with multiplier system  $\chi$  for  $\Gamma_0(N)$ , and  $E_k(\Gamma_0(N), \chi)$  and  $S_k(\Gamma_0(N), \chi)$  to denote the subspaces of Eisenstein forms and cusp forms of  $M_k(\Gamma_0(N), \chi)$ , respectively. We also write  $M_k(\Gamma_0(N))$ ,  $E_k(\Gamma_0(N))$  and  $S_k(\Gamma_0(N))$  for  $M_k(\Gamma_0(N), \chi_1)$ ,  $E_k(\Gamma_0(N), \chi_1)$  and  $S_k(\Gamma_0(N), \chi_1)$ , respectively. It is known (see [12, p. 83]) that

$$(1.3) \quad M_k(\Gamma_0(N), \chi) = E_k(\Gamma_0(N), \chi) \oplus S_k(\Gamma_0(N), \chi).$$

The Dedekind eta function  $\eta(z)$  is the holomorphic function defined on the upper half plane  $\mathbb{H} = \{z \in \mathbb{C} \mid \text{Im}(z) > 0\}$  by

$$(1.4) \quad \eta(z) = e^{\pi iz/12} \prod_{n=1}^{\infty} (1 - e^{2\pi inz}).$$

Throughout the remainder of this paper we set  $q = q(z) := e^{2\pi iz}$  with  $z \in \mathbb{H}$ . Let  $N \in \mathbb{N}$  and  $r_\delta \in \mathbb{Z}$  for each positive divisor  $\delta$  of  $N$ . We define an eta quotient of level  $N$  by the product formula

$$(1.5) \quad f(z) = \prod_{1 \leq \delta|N} \eta^{r_\delta}(\delta z).$$

We define an Eisenstein series  $E_{t_1, t_2}(q)$  by

$$(1.6) \quad E_{t_1, t_2}(z) := C_{t_1, t_2} + \sum_{n=1}^{\infty} \sigma_{(\chi_{t_1}, \chi_{t_2})}(n) q^n$$

for each

$$(t_1, t_2) = (-8, -3), (-3, -8), (1, 24), (24, 1), (1, 1), (1, 8), (8, 1), \\ (1, 12), (12, 1), (-3, -4), (-4, -3),$$

where

$$C_{8,1} = -\frac{1}{2}, \quad C_{12,1} = -1, \quad C_{24,1} = -3, \quad C_{1,1} = -\frac{1}{24}, \\ C_{-8,-3} = C_{-3,-8} = C_{1,24} = C_{1,8} = C_{1,12} = C_{-3,-4} = C_{-4,-3} = 0.$$

For  $(t_1, t_2) = (1, 1)$  we write

$$L(q) := E_{1,1}(z) = -\frac{1}{24} + \sum_{n=1}^{\infty} \sigma(n) q^n.$$

For  $1 < d \mid 24$  we set

$$(1.7) \quad L_d(q) := L(q) - dL(q^d).$$

Ramanujan's theta function  $\varphi(z)$  is defined by

$$\varphi(z) = \sum_{n=-\infty}^{\infty} q^{n^2}.$$

It is well known [5, p. 11] that  $\varphi(z)$  can be expressed as

$$(1.8) \quad \varphi(z) = \frac{\eta^5(2z)}{\eta^2(z)\eta^2(4z)}.$$

Let  $a_1, a_2, a_3, a_4 \in \mathbb{N}$  and  $n \in \mathbb{N}_0$ . We define

$$N(a_1, a_2, a_3, a_4; n) := \text{card}\{(x_1, x_2, x_3, x_4) \in \mathbb{Z}^4 \mid n = a_1x_1^2 + a_2x_2^2 + a_3x_3^2 + a_4x_4^2\}.$$

For notational convenience we set

$$\varphi[a_1, a_2, a_3, a_4](z) := \varphi(a_1z)\varphi(a_2z)\varphi(a_3z)\varphi(a_4z).$$

We have

$$(1.9) \quad \varphi[a_1, a_2, a_3, a_4](z) = \sum_{n=0}^{\infty} N(a_1, a_2, a_3, a_4; n)q^n,$$

which is independent of the order of  $a_1, a_2, a_3, a_4$ . We have 35 theta products  $\varphi[a_1, a_2, a_3, a_4](z)$  of level 24 and weight 2. We group them according to the modular spaces to which they belong, namely

$$(1.10) \quad \begin{cases} \varphi[1, 1, 1, 1](z), & \varphi[1, 1, 2, 2](z), & \varphi[1, 1, 3, 3](z), & \varphi[1, 1, 6, 6](z), \\ \varphi[1, 2, 3, 6](z), & \varphi[2, 2, 2, 2](z), & \varphi[2, 2, 3, 3](z), & \varphi[2, 2, 6, 6](z), \\ \varphi[3, 3, 3, 3](z), & \varphi[3, 3, 6, 6](z), & \varphi[6, 6, 6, 6](z); \end{cases}$$

$$(1.11) \quad \begin{cases} \varphi[1, 1, 1, 2](z), & \varphi[1, 1, 3, 6](z), & \varphi[1, 2, 2, 2](z), & \varphi[1, 2, 3, 3](z), \\ \varphi[1, 2, 6, 6](z), & \varphi[2, 2, 3, 6](z), & \varphi[3, 3, 3, 6](z), & \varphi[3, 6, 6, 6](z); \end{cases}$$

$$(1.12) \quad \begin{cases} \varphi[1, 1, 1, 3](z), & \varphi[1, 1, 2, 6](z), & \varphi[1, 2, 2, 3](z), & \varphi[1, 3, 3, 3](z), \\ \varphi[1, 3, 6, 6](z), & \varphi[2, 2, 2, 6](z), & \varphi[2, 3, 3, 6](z), & \varphi[2, 6, 6, 6](z); \end{cases}$$

$$(1.13) \quad \begin{cases} \varphi[1, 1, 1, 6](z), & \varphi[1, 1, 2, 3](z), & \varphi[1, 2, 2, 6](z), & \varphi[1, 3, 3, 6](z), \\ \varphi[1, 6, 6, 6](z), & \varphi[2, 2, 2, 3](z), & \varphi[2, 3, 3, 3](z), & \varphi[2, 3, 6, 6](z) \end{cases}$$

are in  $M_2(\Gamma_0(24), \chi_1)$ ,  $M_2(\Gamma_0(24), \chi_8)$ ,  $M_2(\Gamma_0(24), \chi_{12})$  and  $M_2(\Gamma_0(24), \chi_{24})$ , respectively.

In this paper we give bases for the spaces  $M_2\left(\Gamma_0(24), \left(\frac{d}{\cdot}\right)\right)$  ( $d = 1, 8, 12, 24$ ) of modular forms. We determine the Fourier coefficients of all 35 theta products in (1.10)–(1.13). We then deduce explicit formulas for  $N(a_1, a_2, a_3, a_4; n)$ , where  $a_1, a_2, a_3, a_4 \in \{1, 2, 3, 6\}$ , in a uniform manner, of which 14 are Ramanujan's universal quaternary quadratic forms given in [11]. We also find all the eta quotients

in the Eisenstein spaces  $E_2\left(\Gamma_0(24), \left(\frac{d}{\cdot}\right)\right)$  ( $d = 1, 8, 12, 24$ ) and give their Fourier coefficients.

## 2. BASES FOR $M_2(\Gamma_0(24), \chi_i)$ FOR $i \in \{1, 8, 12, 24\}$

We deduce from [12, Sec. 6.1, p. 93] that

$$(2.1) \quad \dim(S_2(\Gamma_0(24))) = 1, \dim(E_2(\Gamma_0(40))) = 7.$$

We also deduce from [12, Sec. 6.3, p. 98] that

$$(2.2) \quad \dim(S_2(\Gamma_0(24), \chi_8)) = 2, \dim(S_2(\Gamma_0(24), \chi_{12})) = 0, \dim(S_2(\Gamma_0(24), \chi_{24})) = 2,$$

and

$$(2.3) \quad \dim(E_2(\Gamma_0(24), \chi_8)) = 4, \dim(E_2(\Gamma_0(24), \chi_{12})) = 8, \dim(E_2(\Gamma_0(24), \chi_{24})) = 4.$$

We define the eta quotients

$$(2.4) \quad A(q) := \eta(2z)\eta(4z)\eta(6z)\eta(12z),$$

$$(2.5) \quad B_1(q) := \frac{\eta(z)\eta^4(6z)\eta^2(8z)}{\eta(2z)\eta(3z)\eta(12z)}, \quad B_2(q) := \frac{\eta^2(z)\eta(8z)\eta^4(12z)}{\eta(4z)\eta(6z)\eta(24z)},$$

$$(2.6) \quad C_1(q) := \frac{\eta(z)\eta(4z)\eta^4(6z)\eta^2(24z)}{\eta(2z)\eta(3z)\eta^2(12z)}, \quad C_2(q) := \frac{\eta^2(z)\eta^4(4z)\eta(6z)\eta(24z)}{\eta^2(2z)\eta(8z)\eta(12z)}.$$

We now give a basis for  $M_2(\Gamma_0(24), \chi_i)$  for each  $i \in \{1, 8, 12, 24\}$ .

**Theorem 2.1.** *Let  $\chi_t$  be as in (1.1) for  $t = 1, 8, 12, 24$ . Then*

$$\begin{aligned} & \{L_d(q) \mid d = 2, 3, 4, 6, 8, 12, 24\} \cup \{A(q)\}, \\ & \{E_{1,8}(z), E_{1,8}(3z), E_{8,1}(z), E_{8,1}(3z), B_1(q), B_2(q)\}, \\ & \{E_{1,12}(z), E_{1,12}(2z), E_{12,1}(z), E_{12,1}(2z), E_{-4,-3}(z), \\ & \quad E_{-4,-3}(2z), E_{-3,-4}(z), E_{-3,-4}(2z)\}, \\ & \{E_{1,24}(z), E_{24,1}(z), E_{-3,-8}(z), E_{-8,-3}(z), C_1(q), C_2(q)\}, \end{aligned}$$

are bases for  $M_2(\Gamma_0(24))$ ,  $M_2(\Gamma_0(24), \chi_8)$ ,  $M_2(\Gamma_0(24), \chi_{12})$  and  $M_2(\Gamma_0(24), \chi_{24})$ , respectively.

*Proof.* Appealing to [10, Theorem 1.64, p. 18] and [8, Corollary 2.3, p. 37] (see also [9, 7, 1]) one can show that  $A(q) \in S_2(\Gamma_0(24))$ ;  $B_1(q), B_2(q) \in S_2(\Gamma_0(24), \chi_8)$ ;  $C_1(q), C_2(q) \in S_2(\Gamma_0(24), \chi_{24})$ . The assertion (i) follows from (1.3), (2.1) and [12, Theorem 5.8, p. 88]. (ii) follows from (1.3), (2.2), (2.3) and [12, Theorem 5.9, p. 88] with  $\epsilon = \chi_8$  and  $\chi, \psi \in \{\chi_1, \chi_8\}$ . (iii) follows from (1.3), (2.2) and (2.3) [12, Theorem 5.9, p. 88] with  $\epsilon = \chi_{12}$  and  $\chi, \psi \in \{\chi_1, \chi_{12}, \chi_{-3}, \chi_{-4}\}$ . (iv) follows from (1.3), (2.2), (2.3) and [12, Theorem 5.9, p. 88] with  $\epsilon = \chi_{24}$  and  $\chi, \psi \in \{\chi_1, \chi_{24}, \chi_{-3}, \chi_{-8}\}$ .  $\square$

3. THETA PRODUCTS IN  $M_2(\Gamma_0(24), \chi_i)$  FOR  $i \in \{1, 8, 12, 24\}$ 

Theorems 3.1–3.4 below follow from (1.10)–(1.13) and Theorem 2.1.

**Theorem 3.1.** *Let  $\varphi[a_1, a_2, a_3, a_4](z) \in M_2(\Gamma_0(24))$  be any of the theta products given in (1.10), and let  $L_d(q)$  be as in (1.7). Then we have*

$$\begin{aligned} \varphi[a_1, a_2, a_3, a_4](z) = & b_2 L_2(q) + b_3 L_3(q) + b_4 L_4(q) + b_6 L_6(q) + b_8(L_8(q) \\ & + b_{12} L_{12}(q) + b_{24} L_{24}(q) + x A(q), \end{aligned}$$

where the coefficients  $b_2, b_3, b_4, b_6, b_8, b_{12}, b_{24}$  and  $x$  are given at the right hand side of Table 3.1.

Table 3.1:  $\varphi[a_1, a_2, a_3, a_4](z) = b_2 L_2(q) + b_3 L_3(q) + b_4 L_4(q) + b_6 L_6(q) + b_8(L_8(q) + b_{12} L_{12}(q) + b_{24} L_{24}(q) + x A(q).$

$a_1$	$a_2$	$a_3$	$a_4$	$b_2$	$b_3$	$b_4$	$b_6$	$b_8$	$b_{12}$	$b_{24}$	$x$
1	1	1	1	0	0	8	0	0	0	0	0
1	1	2	2	2	0	−2	0	4	0	0	0
1	1	3	3	4	4	−4	−4	0	4	0	0
1	1	6	6	1	2	1	−1	−2	−1	2	2
1	2	3	6	1/2	−1	−1/2	1/2	1	−1/2	1	1
2	2	2	2	−4	0	0	0	4	0	0	0
2	2	3	3	1	2	1	−1	−2	−1	2	−2
2	2	6	6	−2	0	2	2	−2	−2	2	0
3	3	3	3	0	−8/3	0	0	0	8/3	0	0
3	3	6	6	0	−4/3	0	2/3	0	−2/3	4/3	0
6	6	6	6	0	0	0	−4/3	0	0	4/3	0

**Proof.** Let  $\varphi[a_1, a_2, a_3, a_4](z)$  be any of the theta products listed in (1.10). By Theorem 2.1,  $\varphi[a_1, a_2, a_3, a_4](z)$  must be a linear combination of  $L_d(q)$  ( $d = 2, 3, 4, 6, 8, 12, 24$ ) and  $A(q)$ , namely

$$(3.1) \quad \begin{aligned} \varphi[a_1, a_2, a_3, a_4](z) = & b_2 L_2(q) + b_3 L_3(q) + b_4 L_4(q) + b_6 L_6(q) + b_8(L_8(q) \\ & + b_{12} L_{12}(q) + b_{24} L_{24}(q) + x A(q) \end{aligned}$$

for some scalars  $b_2, b_3, b_4, b_6, b_8, b_{12}, b_{24}, x \in \mathbb{C}$ . By [7, Theorem 3.13], the Sturm bound for the modular space  $M_2(\Gamma_0(24))$  is 8. So, equating the coefficients of  $q^n$  for  $0 \leq n \leq 8$  on both sides of (3.1), we find a system of linear equations. We solve this system and find the asserted coefficients. ■

Similarly to Theorem 3.1, Theorems 3.2–3.4 follow from (1.11)–(1.13) and Theorem 2.1.

**Theorem 3.2.** *Let  $\varphi[a_1, a_2, a_3, a_4](z) \in M_2(\Gamma_0(24), \chi_8)$  be any of the theta products given in (1.11), where  $\chi_8(n)$  is given by (1.1). Let  $E_{1,8}(z)$  and  $E_{8,1}(z)$  be as in (1.6).*

Then we have

$$\begin{aligned} \varphi[a_1, a_2, a_3, a_4](z) &= b_1 E_{1,8}(z) + b_2 E_{1,8}(3z) + b_3 E_{8,1}(z) + b_4 E_{8,1}(3z) \\ &\quad + x_1 B_1(q) + x_2 B_2(q), \end{aligned}$$

where the coefficients  $b_1, b_2, b_3, b_4, x_1$  and  $x_2$  are given at the right hand side of Table 3.2.

Table 3.2:  $\varphi[a_1, a_2, a_3, a_4](z) = b_1 E_{1,8}(z) + b_2 E_{1,8}(3z) + b_3 E_{8,1}(z) + b_4 E_{8,1}(3z) + x_1 B_1(q) + x_2 B_2(q)$

$a_1$	$a_2$	$a_3$	$a_4$	$b_1$	$b_2$	$b_3$	$b_4$	$x_1$	$x_2$
1	1	1	2	8	0	-2	0	0	0
1	1	3	6	16/5	-24/5	-4/5	-6/5	8/5	0
1	2	2	2	4	0	-2	0	0	0
1	2	3	3	8/5	48/5	2/5	-12/5	-8/5	8/5
1	2	6	6	4/5	24/5	2/5	-12/5	8/5	-4/5
2	2	3	6	8/5	-12/5	-4/5	-6/5	0	-4/5
3	3	3	6	0	8	0	-2	0	0
3	6	6	6	0	4	0	-2	0	0

**Theorem 3.3.** Let  $\varphi[a_1, a_2, a_3, a_4](z) \in M_2(\Gamma_0(24), \chi_{12})$  be any of the theta products given in (1.12), where  $\chi_{12}(n)$  is given by (1.1). Let  $E_{1,12}(z)$ ,  $E_{12,1}(z)$ ,  $E_{-3,-4}(z)$  and  $E_{-4,-3}(z)$  be as in (1.6). Then we have

$$\begin{aligned} \varphi[a_1, a_2, a_3, a_4](z) &= b_1 E_{12,1}(z) + b_2 E_{12,1}(2z) + b_3 E_{1,12}(z) + b_4 E_{1,12}(2z) \\ &\quad + b_5 E_{-4,-3}(z) + b_6 E_{-4,-3}(2z) + b_7 E_{-3,-4}(z) + b_8 E_{-3,-4}(2z), \end{aligned}$$

where the coefficients  $b_1, b_2, b_3, b_4, b_5, b_6, b_7$  and  $b_8$  are given at the right hand side of Table 3.3.

Table 3.3:  $\varphi[a_1, a_2, a_3, a_4](z) = b_1 E_{12,1}(z) + b_2 E_{12,1}(2z) + b_3 E_{1,12}(z) + b_4 E_{1,12}(2z) + b_5 E_{-4,-3}(z) + b_6 E_{-4,-3}(2z) + b_7 E_{-3,-4}(z) + b_8 E_{-3,-4}(2z)$ .

$a_1$	$a_2$	$a_3$	$a_4$	$b_1$	$b_2$	$b_3$	$b_4$	$b_5$	$b_6$	$b_7$	$b_8$
1	1	1	3	-1	0	6	0	3	0	-2	0
1	1	2	6	0	-1	3	0	0	3	1	0
1	2	2	3	0	-1	3	0	0	-3	-1	0
1	3	3	3	-1	0	2	0	-1	0	2	0
1	3	6	6	0	-1	1	0	0	1	1	0
2	2	2	6	0	-1	0	6	0	3	0	-2
2	3	3	6	0	-1	1	0	0	-1	-1	0
2	6	6	6	0	-1	0	2	0	-1	0	2

**Theorem 3.4.** *Let  $\varphi[a_1, a_2, a_3, a_4](z) \in M_2(\Gamma_0(24), \chi_{24})$  be any of the theta products given in (1.13), where  $\chi_{24}(n)$  is given by (1.1). Let  $E_{1,24}(z)$ ,  $E_{24,1}(z)$ ,  $E_{-8,-3}(z)$  and  $E_{-3,-8}(z)$  be as in (1.6). Then we have*

$$\begin{aligned} \varphi[a_1, a_2, a_3, a_4](z) = & b_1 E_{24,1}(z) + b_2 E_{1,24}(z) + b_3 E_{-8,-3}(z) + b_4 E_{-3,-8}(z) \\ & + x_1 C_1(q) + x_2 C_2(q), \end{aligned}$$

where the coefficients  $b_1, b_2, b_3, b_4, x_1$  and  $x_2$  are given at the right hand side of Table 3.4.

Table 3.4:  $\varphi[a_1, a_2, a_3, a_4](z) = b_1 E_{24,1}(z) + b_2 E_{1,24}(z) + b_3 E_{-8,-3}(z) + b_4 E_{-3,-8}(z) + x_1 C_1(q) + x_2 C_2(q)$ .

$a_1$	$a_2$	$a_3$	$a_4$	$b_1$	$b_2$	$b_3$	$b_4$	$x_1$	$x_2$
1	1	1	6	$-1/3$	4	1	$-4/3$	8	$8/3$
1	1	2	3	$-1/3$	4	$-1$	$4/3$	0	0
1	2	2	6	$-1/3$	2	1	$-2/3$	0	0
1	3	3	6	$-1/3$	$4/3$	$-1/3$	$4/3$	0	0
1	6	6	6	$-1/3$	$2/3$	$-1/3$	$2/3$	$8/3$	$4/3$
2	2	2	3	$-1/3$	2	$-1$	$2/3$	0	$-4/3$
2	3	3	3	$-1/3$	$4/3$	$1/3$	$-4/3$	$-8/3$	0
2	3	6	6	$-1/3$	$2/3$	$1/3$	$-2/3$	0	0

**Remark 3.1.** *We obtain the following identities from Tables 3.1, 3.2 and 3.4.*

$$\begin{aligned} & -\varphi[1, 1, 2, 2](z) + 4\varphi[1, 2, 3, 6](z) - 3\varphi[3, 3, 6, 6](z) = 4A(q), \\ & -2\varphi[1, 1, 1, 2](z) + 5\varphi[1, 1, 3, 6](z) + 9\varphi[3, 3, 3, 6](z) - 12\varphi[3, 6, 6, 6](z) = 8B_1(q), \\ & 2\varphi[1, 2, 2, 2](z) - 5\varphi[2, 2, 3, 6](z) - 6\varphi[3, 3, 3, 6](z) + 9\varphi[3, 6, 6, 6](z) = 4B_2(q), \\ & 2\varphi[1, 1, 1, 6](z) - 3\varphi[1, 1, 2, 3](z) + 4\varphi[2, 2, 2, 3](z) - 3\varphi[2, 3, 3, 3](z) = 24C_1(q), \\ & 2\varphi[1, 1, 2, 3](z) - 2\varphi[1, 2, 2, 6](z) - 3\varphi[2, 2, 2, 3](z) + 3\varphi[2, 3, 6, 6](z) = 4C_2(q). \end{aligned}$$

#### 4. REPRESENTATIONS BY QUATERNARY QUADRATIC FORMS WITH COEFFICIENTS 1, 2, 3 OR 6

Let  $a_1, a_2, a_3, a_4 \in \{1, 2, 3, 6\}$ . With the simplifying assumptions

$$\gcd(a_1, a_2, a_3, a_4) = 1 \text{ and } a_1 \leq a_2 \leq a_3 \leq a_4,$$

there are 26 diagonal quaternary quadratic forms  $a_1 x_1^2 + a_2 x_2^2 + a_3 x_3^2 + a_4 x_4^2$ . Of these, 14 are Ramanujan's universal quaternary quadratic forms given in [11]. In Theorem 4.1 we give explicit formulas for  $N(a_1, a_2, a_3, a_4; n)$  for these 14 universal quaternary quadratic forms. In Theorem 4.2 we give formulas for  $N(a_1, a_2, a_3, a_4; n)$

for the remaining 12 quaternary quadratic forms. Both Theorems 4.1 and 4.2 follow from Theorems 3.1–3.4.

**Theorem 4.1.** *Let  $n \in \mathbb{N}$ . Then*

- (i)  $N(1, 1, 1, 1; n) = 8\sigma(n) - 32\sigma(n/4)$ ,
- (ii)  $N(1, 1, 2, 2; n) = 4\sigma(n) - 4\sigma(n/2) + 8\sigma(n/4) - 32\sigma(n/8)$ ,
- (iii)  $N(1, 1, 3, 3; n) = 4\sigma(n) - 8\sigma(n/2) - 12\sigma(n/3) + 16\sigma(n/4)$   
 $+ 24\sigma(n/6) - 48\sigma(n/12)$ ,
- (iv)  $N(1, 2, 3, 6; n) = \sigma(n) - \sigma(n/2) + 3\sigma(n/3) + 2\sigma(n/4) - 3\sigma(n/6)$   
 $- 8\sigma(n/8) + 6\sigma(n/12) - 24\sigma(n/24) + a(n)$ ,
- (v)  $N(1, 1, 1, 2; n) = 8\sigma_{(\chi_1, \chi_8)}(n) - 2\sigma_{(\chi_8, \chi_1)}(n)$ ,
- (vi)  $N(1, 1, 3, 6; n) = \frac{16}{5}\sigma_{(\chi_1, \chi_8)}(n) - \frac{24}{5}\sigma_{(\chi_1, \chi_8)}(n/3) - \frac{4}{5}\sigma_{(\chi_8, \chi_1)}(n)$   
 $- \frac{6}{5}\sigma_{(\chi_8, \chi_1)}(n/3) + \frac{8}{5}b_1(n)$ ,
- (vii)  $N(1, 2, 2, 2; n) = 4\sigma_{(\chi_1, \chi_8)}(n) - 2\sigma_{(\chi_8, \chi_1)}(n)$ ,
- (viii)  $N(1, 2, 3, 3; n) = \frac{8}{5}\sigma_{(\chi_1, \chi_8)}(n) + \frac{48}{5}\sigma_{(\chi_1, \chi_8)}(n/3) + \frac{2}{5}\sigma_{(\chi_8, \chi_1)}(n)$   
 $- \frac{12}{5}\sigma_{(\chi_8, \chi_1)}(n/3) - \frac{8}{5}b_1(n) + \frac{8}{5}b_2(n)$ ,
- (ix)  $N(1, 1, 1, 3; n) = 6\sigma_{(\chi_1, \chi_{12})}(n) - \sigma_{(\chi_{12}, \chi_1)}(n) - 2\sigma_{(\chi_{-3}, \chi_{-4})}(n) + 3\sigma_{(\chi_{-4}, \chi_{-3})}(n)$ ,
- (x)  $N(1, 1, 2, 6; n) = 3\sigma_{(\chi_1, \chi_{12})}(n) - \sigma_{(\chi_{12}, \chi_1)}(n/2) + \sigma_{(\chi_{-3}, \chi_{-4})}(n) + 3\sigma_{(\chi_{-4}, \chi_{-3})}(n/2)$ ,
- (xi)  $N(1, 2, 2, 3; n) = 3\sigma_{(\chi_1, \chi_{12})}(n) - \sigma_{(\chi_{12}, \chi_1)}(n/2) - \sigma_{(\chi_{-3}, \chi_{-4})}(n) - 3\sigma_{(\chi_{-4}, \chi_{-3})}(n/2)$ ,
- (xii)  $N(1, 1, 1, 6; n) = 4\sigma_{(\chi_1, \chi_{24})}(n) - \frac{1}{3}\sigma_{(\chi_{24}, \chi_1)}(n) - \frac{4}{3}\sigma_{(\chi_{-3}, \chi_{-8})}(n)$   
 $+ \sigma_{(\chi_{-8}, \chi_{-3})}(n) + 8c_1(n) + \frac{8}{3}c_2(n)$ ,
- (xiii)  $N(1, 1, 2, 3; n) = 4\sigma_{(\chi_1, \chi_{24})}(n) - \frac{1}{3}\sigma_{(\chi_{24}, \chi_1)}(n) + \frac{4}{3}\sigma_{(\chi_{-3}, \chi_{-8})}(n) - \sigma_{(\chi_{-8}, \chi_{-3})}(n)$ ,
- (xiv)  $N(1, 2, 2, 6; n) = 2\sigma_{(\chi_1, \chi_{24})}(n) - \frac{1}{3}\sigma_{(\chi_{24}, \chi_1)}(n) - \frac{2}{3}\sigma_{(\chi_{-3}, \chi_{-8})}(n) + \sigma_{(\chi_{-8}, \chi_{-3})}(n)$ .

**Theorem 4.2.** *Let  $n \in \mathbb{N}$ . Then*

- (i)  $N(1, 1, 6, 6; n) = 2\sigma(n) - 2\sigma(n/2) - 6\sigma(n/3) - 4\sigma(n/4) + 6\sigma(n/6)$   
 $+ 16\sigma(n/8) + 12\sigma(n/12) - 48\sigma(n/24) + 2a(n)$ ,
- (ii)  $N(2, 2, 3, 3; n) = 4\sigma(n) - 8\sigma(n/2) - 12\sigma(n/3) + 16\sigma(n/4)$   
 $+ 24\sigma(n/6) - 48\sigma(n/12)$ ,



$$\begin{aligned}
 \text{(iii)} \quad N(1, 2, 6, 6; n) &= \frac{4}{5}\sigma_{(\chi_1, \chi_8)}(n) + \frac{24}{5}\sigma_{(\chi_1, \chi_8)}(n/3) + \frac{2}{5}\sigma_{(\chi_8, \chi_1)}(n) \\
 &\quad - \frac{12}{5}\sigma_{(\chi_8, \chi_1)}(n/3) + \frac{8}{5}b_1(n) - \frac{4}{5}b_2(n), \\
 \text{(iv)} \quad N(2, 2, 3, 6; n) &= \frac{8}{5}\sigma_{(\chi_1, \chi_8)}(n) - \frac{12}{5}\sigma_{(\chi_1, \chi_8)}(n/3) - \frac{4}{5}\sigma_{(\chi_8, \chi_1)}(n) \\
 &\quad - \frac{6}{5}\sigma_{(\chi_8, \chi_1)}(n/3) - \frac{4}{5}b_2(n), \\
 \text{(v)} \quad N(1, 3, 3, 3; n) &= 2\sigma_{(\chi_1, \chi_{12})}(n) - \sigma_{(\chi_{12}, \chi_1)}(n) + 2\sigma_{(\chi_{-3}, \chi_{-4})}(n) - \sigma_{(\chi_{-4}, \chi_{-3})}(n), \\
 \text{(vi)} \quad N(1, 3, 6, 6; n) &= \sigma_{(\chi_1, \chi_{12})}(n) - \sigma_{(\chi_{12}, \chi_1)}(n/2) + \sigma_{(\chi_{-3}, \chi_{-4})}(n) + \sigma_{(\chi_{-4}, \chi_{-3})}(n/2), \\
 \text{(vii)} \quad N(2, 3, 3, 6; n) &= \sigma_{(\chi_1, \chi_{12})}(n) - \sigma_{(\chi_{12}, \chi_1)}(n/2) - \sigma_{(\chi_{-3}, \chi_{-4})}(n) - \sigma_{(\chi_{-4}, \chi_{-3})}(n/2), \\
 \text{(viii)} \quad N(1, 3, 3, 6; n) &= \frac{4}{3}\sigma_{(\chi_1, \chi_{24})}(n) - \frac{1}{3}\sigma_{(\chi_{24}, \chi_1)}(n) + \frac{4}{3}\sigma_{(\chi_{-3}, \chi_{-8})}(n) - \frac{1}{3}\sigma_{(\chi_{-8}, \chi_{-3})}(n), \\
 \text{(ix)} \quad N(1, 6, 6, 6; n) &= \frac{2}{3}\sigma_{(\chi_1, \chi_{24})}(n) - \frac{1}{3}\sigma_{(\chi_{24}, \chi_1)}(n) + \frac{2}{3}\sigma_{(\chi_{-3}, \chi_{-8})}(n) \\
 &\quad - \frac{1}{3}\sigma_{(\chi_{-8}, \chi_{-3})}(n) + \frac{8}{3}c_1(n) + \frac{4}{3}c_2(n), \\
 \text{(x)} \quad N(2, 2, 2, 3; n) &= 2\sigma_{(\chi_1, \chi_{24})}(n) - \frac{1}{3}\sigma_{(\chi_{24}, \chi_1)}(n) + \frac{2}{3}\sigma_{(\chi_{-3}, \chi_{-8})}(n) \\
 &\quad - \sigma_{(\chi_{-8}, \chi_{-3})}(n) - \frac{4}{3}c_2(n), \\
 \text{(xi)} \quad N(2, 3, 3, 3; n) &= \frac{4}{3}\sigma_{(\chi_1, \chi_{24})}(n) - \frac{1}{3}\sigma_{(\chi_{24}, \chi_1)}(n) - \frac{4}{3}\sigma_{(\chi_{-3}, \chi_{-8})}(n) \\
 &\quad + \frac{1}{3}\sigma_{(\chi_{-8}, \chi_{-3})}(n) - \frac{8}{3}c_1(n), \\
 \text{(xii)} \quad N(2, 3, 6, 6; n) &= \frac{1}{3}(2\sigma_{(\chi_1, \chi_{24})}(n) - \sigma_{(\chi_{24}, \chi_1)}(n) - 2\sigma_{(\chi_{-3}, \chi_{-8})}(n) + \sigma_{(\chi_{-8}, \chi_{-3})}(n)).
 \end{aligned}$$

**Remark 4.1.** The formula in Theorem 4.1(i) is the classical result of Jacobi [6, 14]. The formulas in Theorems 4.1(xiii)(xiv) and 4.2(viii)(xii) agree with the formulas given in [4, p. 1668]. Note that  $A(n)$ ,  $B(n)$ ,  $C(n)$  and  $D(n)$  in [4] are  $\sigma_{(\chi_1, \chi_{24})}(n)$ ,  $\sigma_{(\chi_{-8}, \chi_{-3})}(n)$ ,  $\sigma_{(\chi_{-3}, \chi_{-8})}(n)$  and  $\sigma_{(\chi_{24}, \chi_1)}(n)$  in this paper, respectively. The formulas in Theorem 4.1(v)(vii) agree with those given in [13]. The formulas in Theorems 4.1(i)–(iv) and 4.2(i)(ii) agree with those given in [3].

## 5. ETA QUOTIENTS IN $E_2(\Gamma_0(24), \chi_i)$ FOR $i \in \{1, 8, 12, 24\}$

Using MAPLE, we found that there are exactly 819, 212, 800 and 212 eta quotients in  $M_2(\Gamma_0(24), \chi_i)$  for  $i \in \{1, 8, 12, 24\}$ , respectively. Of these, 282, 8, 800 and 8 eta quotients are in  $E_2(\Gamma_0(24), \chi_i)$  for  $i \in \{1, 8, 12, 24\}$ , respectively. We note that as  $\dim(S_2(\Gamma_0(24), \chi_{12})) = 0$ , we have  $M_2(\Gamma_0(24), \chi_{12}) = E_2(\Gamma_0(24), \chi_{12})$ .

Of the eta quotients in  $E_2(\Gamma_0(24))$ , 250 arise directly from those given in [15]. In Table 5.1 we list the remaining 32 eta quotients and their Fourier series expansions.

In Table 5.2 we list all 8 eta quotients in  $E_2(\Gamma_0(24), \chi_{24})$ . All 8 eta quotients in  $E_2(\Gamma_0(24), \chi_8)$  arise directly from those in  $E_2(\Gamma_0(8), \chi_8)$  given in [2], so we do not list them here.

All the eta quotients in  $E_2(\Gamma_0(12), \chi_{12})$  are given in [1]. There are 395 eta quotients in  $E_2(\Gamma_0(24), \chi_{12})$ , which do not arise directly from those in  $E_2(\Gamma_0(12), \chi_{12})$ . We list these eta quotients and their Fourier series expansions in Table 5.3.

Theorems 5.1–5.3 follow from Theorem 2.1 directly.

**Theorem 5.1.** *Let  $f(z) = \prod_{1 \leq \delta | 24} \eta^{r_\delta}(\delta z) \in E_2(\Gamma_0(24))$  be any of the eta quotients*

*with the exponents  $r_\delta$  given on the left hand side of Table 5.1. Then we have*

$$f(z) = b_2 L_2(q) + b_3 L_3(q) + b_4 L_4(q) + b_6 L_6(q) + b_8 L_8(q) + b_{12} L_{12}(q) + b_{24} L_{24}(q),$$

*where the coefficients  $b_j$  ( $j \in \{2, 3, 4, 6, 8, 12, 24\}$ ) are given at the right hand side of Table 5.1.*

Table 5.1:  $f(z) = b_2 L_2(q) + b_3 L_3(q) + b_4 L_4(q) + b_6 L_6(q) + b_8 L_8(q) + b_{12} L_{12}(q) + b_{24} L_{24}(q)$ .

$r_1$	$r_2$	$r_3$	$r_4$	$r_6$	$r_8$	$r_{12}$	$r_{24}$	$b_2$	$b_3$	$b_4$	$b_6$	$b_8$	$b_{12}$	$b_{24}$
-1	4	-1	-5	-2	2	9	-2	-1/2	-1/3	5/4	1/3	-1/2	-5/12	1/6
-1	2	-1	1	0	-2	3	2	0	-1/3	-1/4	1/6	1/2	1/12	-1/6
-1	0	-1	3	2	2	1	-2	1/2	0	1/4	0	-1/2	-3/4	3/2
-1	-2	-1	9	4	-2	-5	2	1	0	-5/4	-3/2	1/2	15/4	-3/2
-2	5	2	-4	-5	1	6	1	0	-1/3	-1/2	5/6	1/4	-1/6	-1/12
-2	4	2	-1	-6	-1	9	-1	-1/2	-2/3	1/2	11/6	-1/2	-5/6	1/6
-2	1	2	4	-1	1	-2	1	1/2	0	-1/2	0	1/4	3/2	-3/4
-2	0	2	7	-2	-1	1	-1	1/2	0	1/2	3/2	-1/2	-3/2	3/2
-1	1	-1	-2	7	2	0	-2	1	1/3	-1	-1/3	0	-1/3	4/3
-1	-1	-1	4	9	-2	-6	2	1	1/3	-1	-5/3	0	11/3	-4/3
-2	2	2	-1	4	1	-3	1	1/2	1/3	0	-5/6	0	5/3	-2/3
-2	1	2	2	3	-1	0	-1	1	2/3	0	-1/3	0	-2/3	4/3
2	-1	-2	-2	1	1	4	1	0	-1/3	1/2	1/6	-1/4	-1/6	1/12
2	-2	-2	1	0	-1	7	-1	-1/2	2/3	1/2	-1/6	-1/2	-1/6	1/6
2	-5	-2	6	5	1	-4	1	5/2	0	-1/2	0	-1/4	-3/2	3/4
2	-6	-2	9	4	-1	-1	-1	-11/2	0	5/2	3/2	-1/2	-3/2	3/2
1	1	1	-4	-5	2	10	-2	-1/2	1/3	5/4	-2/3	-1/2	-1/12	1/6
1	-1	1	2	-3	-2	4	2	0	-1/3	1/4	5/6	-1/2	-5/12	1/6
1	-3	1	4	-1	2	2	-2	-5/2	0	5/4	0	-1/2	-3/4	3/2
1	-5	1	10	1	-2	-4	2	2	0	1/4	3/2	-1/2	-15/4	3/2
2	-4	-2	1	10	1	-5	1	5/2	1/3	-1	-1/6	0	-4/3	2/3
2	-5	-2	4	9	-1	-2	-1	-5	-2/3	2	5/3	0	-4/3	4/3
1	-2	1	-1	4	2	1	-2	-2	-1/3	0	2/3	0	-2/3	4/3
1	-4	1	5	6	-2	-5	2	2	1/3	0	2/3	0	-10/3	4/3
-1	9	-1	-6	-1	2	4	-2	5	3	-11	-3	4	3	0
-1	7	-1	0	1	-2	-2	2	1	3	1	-3	-4	3	0
-2	10	2	-5	-4	1	1	1	1/2	3	4	-15/2	-2	3	0

$r_1$	$r_2$	$r_3$	$r_4$	$r_6$	$r_8$	$r_{12}$	$r_{24}$	$b_2$	$b_3$	$b_4$	$b_6$	$b_8$	$b_{12}$	$b_{24}$
-2	9	2	-2	-5	-1	4	-1	5	6	-4	-15	4	6	0
2	4	-2	-3	2	1	-1	1	5/2	3	-5	-3/2	2	0	0
2	3	-2	0	1	-1	2	-1	-1	-6	-2	3	4	0	0
1	6	1	-5	-4	2	5	-2	2	-3	-10	6	4	0	0
1	4	1	1	-2	-2	-1	2	2	3	-2	-6	4	0	0

**Theorem 5.2.** Let  $f(z) = \prod_{1 \leq \delta | 24} \eta^{r_\delta}(\delta z) \in E_2(\Gamma_0(24), \chi_{24})$  be any of the eta quotients with the exponents  $r_\delta$  given on the left hand side of Table 5.2, where  $\chi_{24}(n)$  is given by (1.1). Then we have

$$f(z) = b_1 E_{24,1}(z) + b_2 E_{1,24}(z) + b_3 E_{-8,-3}(z) + b_4 E_{-3,-8}(z),$$

where the coefficients  $b_1, b_2, b_3, b_4$  are given at the right hand side of Table 5.2.

Table 5.2:  $f(z) = b_1 E_{24,1}(z) + b_2 E_{1,24}(z) + b_3 E_{-8,-3}(z) + b_4 E_{-3,-8}(z)$ .

$r_1$	$r_2$	$r_3$	$r_4$	$r_6$	$r_8$	$r_{12}$	$r_{24}$	$b_1$	$b_2$	$b_3$	$b_4$
0	-2	-2	5	1	-2	8	-4	-1/3	2/3	1/3	-2/3
-1	-1	1	6	-2	-3	5	-1	0	2/3	1/3	0
-2	1	0	8	-2	-4	5	-2	-1/3	2	1	-2/3
-2	5	-4	-2	8	0	1	-2	-1/3	4/3	-1/3	4/3
-3	6	-1	-1	5	-1	-2	1	0	4/3	-1/3	0
-4	8	-2	1	5	-2	-2	0	-1/3	4	-1	4/3
1	-2	-1	5	-1	-1	6	-3	-1/3	0	0	-2/3
-1	5	-3	-2	6	1	-1	-1	-1/3	0	0	4/3

**Theorem 5.3.** Let  $f(z) = \prod_{1 \leq \delta | 24} \eta^{r_\delta}(\delta z) \in E_2(\Gamma_0(24), \chi_{12})$  be any of the eta quotients with the exponents  $r_\delta$  given on the left hand side of Table 5.3, where  $\chi_{12}(n)$  is given by (1.1). Then we have

$$f(z) = b_1 E_{12,1}(z) + b_2 E_{12,1}(2z) + b_3 E_{1,12}(z) + b_4 E_{1,12}(2z) + b_5 E_{-4,-3}(z) \\ + b_6 E_{-4,-3}(2z) + b_7 E_{-3,-4}(z) + b_8 E_{-3,-4}(2z),$$

where the coefficients  $b_i$  ( $1 \leq i \leq 8$ ) are given at the right hand side of Table 5.3.

Table 5.3:  $f(z) = b_1 E_{12,1}(z) + b_2 E_{12,1}(2z) + b_3 E_{1,12}(z) + b_4 E_{1,12}(2z) + b_5 E_{-4,-3}(z) + b_6 E_{-4,-3}(2z) + b_7 E_{-3,-4}(z) + b_8 E_{-3,-4}(2z)$

$r_1$	$r_2$	$r_3$	$r_4$	$r_6$	$r_8$	$r_{12}$	$r_{24}$	$b_1$	$b_2$	$b_3$	$b_4$	$b_5$	$b_6$	$b_7$	$b_8$
0	0	0	1	0	-2	-5	10	-1/16	1/16	1/48	-1/24	-1/48	-1/48	1/16	1/8
0	1	0	-2	-3	0	4	4	0	0	1/24	-1/12	1/12	1/12	-1/8	-1/4
0	2	0	-5	-6	2	13	-2	0	0	1/12	-1/6	-1/3	-1/3	1/4	1/2
0	3	0	-8	-9	4	22	-8	0	0	1/6	-1/3	4/3	4/3	-1/2	-1
0	0	0	-1	0	2	-3	6	-1/16	1/16	1/16	-1/8	1/16	1/16	-1/16	-1/8
0	1	0	-4	-3	4	6	0	0	0	1/8	-1/4	-1/4	-1/4	1/8	1/4
0	2	0	-7	-6	6	15	-6	0	0	1/4	-1/2	1	1	-1/4	-1/2
0	0	0	-3	0	6	-1	2	-1/16	1/16	3/16	-3/8	-3/16	-3/16	1/16	1/8
0	1	0	-6	-3	8	8	-4	0	0	3/8	-3/4	3/4	3/4	-1/8	-1/4
0	0	0	-5	0	10	1	-2	-1/16	1/16	9/16	-9/8	9/16	9/16	-1/16	-1/8
1	-2	-3	0	6	0	-2	4	0	0	-1/24	1/3	-1/12	-1/12	1/8	0
1	-1	-3	-3	3	2	7	-2	0	0	-1/12	2/3	1/3	1/3	-1/4	0
1	0	-3	-6	0	4	16	-8	0	-1	-1/6	4/3	-4/3	-4/3	1/2	0
1	-2	-3	-2	6	4	0	0	0	0	-1/8	1	1/4	1/4	-1/8	0
1	-1	-3	-5	3	6	9	-6	0	-1	-1/4	2	-1	-1	1/4	0
1	-2	-3	-4	6	8	2	-4	0	-1	-3/8	3	-3/4	-3/4	1/8	0
2	-5	-6	2	15	0	-8	4	-1/4	1/4	5/12	-4/3	1/12	1/12	-1/4	0
2	-4	-6	-1	12	2	1	-2	0	0	5/6	-8/3	-1/3	-1/3	1/2	0
2	-5	-6	0	15	4	-6	0	-1/4	1/4	5/4	-4	-1/4	-1/4	1/4	0
3	-7	-9	1	21	2	-5	-2	0	-1	-7/3	32/3	1/3	1/3	-1	0
1	-4	-3	6	8	-4	-8	8	-1/4	1/4	1/8	-1/3	0	1/3	1/8	0
1	-3	-3	3	5	-2	1	2	0	0	1/4	-2/3	0	-1/3	-1/4	0
1	-2	-3	0	2	0	10	-4	0	0	1/2	-4/3	0	1/3	1/2	0
1	-4	-3	4	8	0	-6	4	-1/4	1/4	3/8	-1	0	0	-1/8	0
1	-3	-3	1	5	2	3	-2	0	0	3/4	-2	0	0	1/4	0
1	-4	-3	2	8	4	-4	0	-1/4	1/4	9/8	-3	0	0	1/8	0
2	-6	-6	5	14	-2	-5	2	0	0	-1/2	8/3	0	1/3	1/2	0
2	-5	-6	2	11	0	4	-4	0	-1	-1	16/3	0	-1/3	-1	0
2	-6	-6	3	14	2	-3	-2	0	-1	-3/2	8	0	0	-1/2	0
3	-9	-9	7	23	-2	-11	2	-1	1	3	-32/3	0	-1/3	-1	0
0	-3	0	8	1	-4	-6	8	-1/4	1/4	1/8	-1/4	0	0	1/8	1/4
0	-2	0	5	-2	-2	3	2	0	0	1/4	-1/2	0	0	-1/4	-1/2
0	-1	0	2	-5	0	12	-4	0	0	1/2	-1	0	0	1/2	1
0	-3	0	6	1	0	-4	4	-1/4	1/4	3/8	-3/4	0	0	-1/8	-1/4
0	-2	0	3	-2	2	5	-2	0	0	3/4	-3/2	0	0	1/4	1/2
0	-3	0	4	1	4	-2	0	-1/4	1/4	9/8	-9/4	0	0	1/8	1/4
1	-5	-3	7	7	-2	-3	2	0	0	-1/4	2	0	0	1/4	0
1	-4	-3	4	4	0	6	-4	0	-1	-1/2	4	0	0	-1/2	0
1	-5	-3	5	7	2	-1	-2	0	-1	-3/4	6	0	0	-1/4	0
2	-8	-6	9	16	-2	-9	2	-1	1	5/2	-8	0	0	-1/2	0
1	-7	-3	13	9	-6	-9	6	-1	1	3/4	-2	0	1	1/4	0

$r_1$	$r_2$	$r_3$	$r_4$	$r_6$	$r_8$	$r_{12}$	$r_{24}$	$b_1$	$b_2$	$b_3$	$b_4$	$b_5$	$b_6$	$b_7$	$b_8$
1	-6	-3	10	6	-4	0	0	0	0	3/2	-4	0	-1	-1/2	0
1	-7	-3	11	9	-2	-7	2	-1	1	9/4	-6	0	0	-1/4	0
2	-9	-6	12	15	-4	-6	0	0	-1	-3	16	0	1	1	0
0	-6	0	15	2	-6	-7	6	-1	1	3/4	-3/2	0	0	1/4	1/2
0	-5	0	12	-1	-4	2	0	0	0	3/2	-3	0	0	-1/2	-1
0	-6	0	13	2	-2	-5	2	-1	1	9/4	-9/2	0	0	-1/4	-1/2
1	-8	-3	14	8	-4	-4	0	0	-1	-3/2	12	0	0	1/2	0
1	-10	-3	20	10	-8	-10	4	-4	4	9/2	-12	0	3	1/2	0
0	-9	0	22	3	-8	-8	4	-4	4	9/2	-9	0	0	1/2	1
0	-1	0	1	3	-1	-5	7	-1/8	1/8	1/12	-1/6	1/24	1/24	0	0
0	0	0	-2	0	1	4	1	0	0	1/6	-1/3	-1/6	-1/6	0	0
0	1	0	-5	-3	3	13	-5	0	0	1/3	-2/3	2/3	2/3	0	0
0	-1	0	-1	3	3	-3	3	-1/8	1/8	1/4	-1/2	-1/8	-1/8	0	0
0	0	0	-4	0	5	6	-3	0	0	1/2	-1	1/2	1/2	0	0
0	-1	0	-3	3	7	-1	-1	-1/8	1/8	3/4	-3/2	3/8	3/8	0	0
1	-3	-3	0	9	1	-2	1	0	0	-1/6	4/3	1/6	1/6	0	0
1	-2	-3	-3	6	3	7	-5	0	-1	-1/3	8/3	-2/3	-2/3	0	0
1	-3	-3	-2	9	5	0	-3	0	-1	-1/2	4	-1/2	-1/2	0	0
2	-6	-6	2	18	1	-8	1	-1/2	1/2	5/3	-16/3	-1/6	-1/6	0	0
1	-5	-3	6	11	-3	-8	5	-1/2	1/2	1/2	-4/3	0	1/3	0	0
1	-4	-3	3	8	-1	1	-1	0	0	1	-8/3	0	-1/3	0	0
1	-5	-3	4	11	1	-6	1	-1/2	1/2	3/2	-4	0	0	0	0
2	-7	-6	5	17	-1	-5	-1	0	-1	-2	32/3	0	1/3	0	0
0	-4	0	8	4	-3	-6	5	-1/2	1/2	1/2	-1	0	0	0	0
0	-3	0	5	1	-1	3	-1	0	0	1	-2	0	0	0	0
0	-4	0	6	4	1	-4	1	-1/2	1/2	3/2	-3	0	0	0	0
1	-6	-3	7	10	-1	-3	-1	0	-1	-1	8	0	0	0	0
1	-8	-3	13	12	-5	-9	3	-2	2	3	-8	0	1	0	0
0	-7	0	15	5	-5	-7	3	-2	2	3	-6	0	0	0	0
-1	1	3	-1	-3	0	1	4	0	0	1/24	1/6	1/12	1/12	-1/8	-1/2
-1	2	3	-4	-6	2	10	-2	0	0	1/12	1/3	-1/3	-1/3	1/4	1
-1	3	3	-7	-9	4	19	-8	0	-1	1/6	2/3	4/3	4/3	-1/2	-2
-1	1	3	-3	-3	4	3	0	0	0	1/8	1/2	-1/4	-1/4	1/8	1/2
-1	2	3	-6	-6	6	12	-6	0	-1	1/4	1	1	1	-1/4	-1
-1	1	3	-5	-3	8	5	-4	0	-1	3/8	3/2	3/4	3/4	-1/8	-1/2
0	-2	0	1	6	0	-5	4	-1/4	1/4	1/3	-2/3	-1/12	-1/12	0	0
0	-1	0	-2	3	2	4	-2	0	0	2/3	-4/3	1/3	1/3	0	0
0	-2	0	-1	6	4	-3	0	-1/4	1/4	1	-2	1/4	1/4	0	0
1	-4	-3	0	12	2	-2	-2	0	-1	-2/3	16/3	-1/3	-1/3	0	0
-1	-1	3	5	-1	-4	-5	8	-1/4	1/4	1/8	-1/6	0	-1/3	1/8	1/2
-1	0	3	2	-4	-2	4	2	0	0	1/4	-1/3	0	1/3	-1/4	-1
-1	1	3	-1	-7	0	13	-4	0	0	1/2	-2/3	0	-1/3	1/2	2
-1	-1	3	3	-1	0	-3	4	-1/4	1/4	3/8	-1/2	0	0	-1/8	-1/2
-1	0	3	0	-4	2	6	-2	0	0	3/4	-1	0	0	1/4	1
-1	-1	3	1	-1	4	-1	0	-1/4	1/4	9/8	-3/2	0	0	1/8	1/2
1	-6	-3	6	14	-2	-8	2	-1	1	2	-16/3	0	1/3	0	0

$r_1$	$r_2$	$r_3$	$r_4$	$r_6$	$r_8$	$r_{12}$	$r_{24}$	$b_1$	$b_2$	$b_3$	$b_4$	$b_5$	$b_6$	$b_7$	$b_8$
-1	-2	3	6	-2	-2	0	2	0	0	1/4	1	0	0	-1/4	-1
-1	-1	3	3	-5	0	9	-4	0	-1	1/2	2	0	0	1/2	2
-1	-2	3	4	-2	2	2	-2	0	-1	3/4	3	0	0	1/4	1
0	-5	0	8	7	-2	-6	2	-1	1	2	-4	0	0	0	0
-1	-4	3	12	0	-6	-6	6	-1	1	3/4	-1	0	-1	1/4	1
-1	-3	3	9	-3	-4	3	0	0	0	3/2	-2	0	1	-1/2	-2
-1	-4	3	10	0	-2	-4	2	-1	1	9/4	-3	0	0	-1/4	-1
-1	-5	3	13	-1	-4	-1	0	0	-1	3/2	6	0	0	-1/2	-2
-1	-7	3	19	1	-8	-7	4	-4	4	9/2	-6	0	-3	1/2	2
-1	0	3	-1	0	1	1	1	0	0	1/6	2/3	-1/6	-1/6	0	0
-1	1	3	-4	-3	3	10	-5	0	-1	1/3	4/3	2/3	2/3	0	0
-1	0	3	-3	0	5	3	-3	0	-1	1/2	2	1/2	1/2	0	0
0	-3	0	1	9	1	-5	1	-1/2	1/2	4/3	-8/3	1/6	1/6	0	0
-1	-2	3	5	2	-3	-5	5	-1/2	1/2	1/2	-2/3	0	-1/3	0	0
-1	-1	3	2	-1	-1	4	-1	0	0	1	-4/3	0	1/3	0	0
-1	-2	3	3	2	1	-3	1	-1/2	1/2	3/2	-2	0	0	0	0
-1	-3	3	6	1	-1	0	-1	0	-1	1	4	0	0	0	0
-1	-5	3	12	3	-5	-6	3	-2	2	3	-4	0	-1	0	0
-2	1	6	0	-3	0	-2	4	-1/4	1/4	5/12	-1/3	1/12	1/12	-1/4	-1
-2	2	6	-3	-6	2	7	-2	0	0	5/6	-2/3	-1/3	-1/3	1/2	2
-2	1	6	-2	-3	4	0	0	-1/4	1/4	5/4	-1	-1/4	-1/4	1/4	1
-1	-1	3	-1	3	2	1	-2	0	-1	2/3	8/3	1/3	1/3	0	0
-2	0	6	3	-4	-2	1	2	0	0	1/2	2/3	0	1/3	-1/2	-2
-2	1	6	0	-7	0	10	-4	0	-1	1	4/3	0	-1/3	1	4
-2	0	6	1	-4	2	3	-2	0	-1	3/2	2	0	0	1/2	2
-1	-3	3	5	5	-2	-5	2	-1	1	2	-8/3	0	-1/3	0	0
-2	-2	6	7	-2	-2	-3	2	-1	1	5/2	-2	0	0	-1/2	-2
-2	-3	6	10	-3	-4	0	0	0	-1	3	4	0	1	-1	-4
-2	0	6	0	0	1	-2	1	-1/2	1/2	5/3	-4/3	-1/6	-1/6	0	0
-2	-1	6	3	-1	-1	1	-1	0	-1	2	8/3	0	1/3	0	0
-3	2	9	-2	-6	2	4	-2	0	-1	7/3	4/3	-1/3	-1/3	1	4
-3	0	9	4	-4	-2	-2	2	-1	1	3	-4/3	0	1/3	-1	-4
-1	3	-1	-1	1	-2	-1	6	0	0	1/24	0	1/12	-1/4	-1/8	0
-1	5	-1	-7	-5	2	17	-6	0	0	1/6	0	4/3	1	-1/2	0
-1	3	-1	-3	1	2	1	2	0	0	1/8	0	-1/4	-1/4	1/8	0
-1	4	-1	-6	-2	4	10	-4	0	0	1/4	0	1	1	-1/4	0
-1	3	-1	-5	1	6	3	-2	0	0	3/8	0	3/4	3/4	-1/8	0
0	0	-4	1	10	-2	-7	6	-1/4	1/4	1/12	0	-1/12	1/4	1/4	0
0	2	-4	-5	4	2	11	-6	0	-1	1/3	0	-4/3	-1	1	0
0	0	-4	-1	10	2	-5	2	-1/4	1/4	1/4	0	1/4	1/4	-1/4	0
0	1	-4	-4	7	4	4	-4	0	-1	1/2	0	-1	-1	1/2	0
0	0	-4	-3	10	6	-3	-2	-1/4	-3/4	3/4	0	-3/4	-3/4	1/4	0
-1	1	-1	3	3	-2	-5	6	-1/4	1/4	1/8	0	0	0	1/8	0
-1	3	-1	-3	-3	2	13	-6	0	-1	1/2	0	0	0	1/2	0
-1	1	-1	1	3	2	-3	2	-1/4	1/4	3/8	0	0	0	-1/8	0

$r_1$	$r_2$	$r_3$	$r_4$	$r_6$	$r_8$	$r_{12}$	$r_{24}$	$b_1$	$b_2$	$b_3$	$b_4$	$b_5$	$b_6$	$b_7$	$b_8$
-1	2	-1	-2	0	4	6	-4	0	-1	3/4	0	0	0	1/4	0
-1	1	-1	-1	3	6	-1	-2	-1/4	-3/4	9/8	0	0	0	1/8	0
-1	0	-1	6	2	-4	-2	4	0	0	1/4	0	0	-1	-1/4	0
-1	1	-1	3	-1	-2	7	-2	0	0	1/2	0	0	1	1/2	0
0	-3	-4	8	11	-4	-8	4	-1	1	1/2	0	0	1	1/2	0
0	-2	-4	5	8	-2	1	-2	0	-1	1	0	0	-1	-1	0
-1	-2	-1	10	4	-4	-6	4	-1	1	3/4	0	0	0	1/4	0
-1	-1	-1	7	1	-2	3	-2	0	-1	3/2	0	0	0	-1/2	0
-1	-3	-1	13	3	-6	-3	2	0	0	3/2	0	0	-3	-1/2	0
0	-6	-4	15	12	-6	-9	2	-4	3	3	0	0	3	1	0
-1	-5	-1	17	5	-6	-7	2	-4	3	9/2	0	0	0	1/2	0
-2	4	2	0	-2	-3	-4	9	-1/8	1/8	1/24	0	-1/24	-3/8	1/8	1/2
-2	5	2	-3	-5	-1	5	3	0	0	1/12	0	1/6	1/2	-1/4	-1
-2	6	2	-6	-8	1	14	-3	0	0	1/6	0	-2/3	-1	1/2	2
-2	7	2	-9	-11	3	23	-9	0	-1	1/3	0	8/3	3	-1	-4
-2	4	2	-2	-2	1	-2	5	-1/8	1/8	1/8	0	1/8	1/8	-1/8	-1/2
-2	5	2	-5	-5	3	7	-1	0	0	1/4	0	-1/2	-1/2	1/4	1
-2	6	2	-8	-8	5	16	-7	0	-1	1/2	0	2	2	-1/2	-2
-2	4	2	-4	-2	5	0	1	-1/8	1/8	3/8	0	-3/8	-3/8	1/8	1/2
-2	5	2	-7	-5	7	9	-5	0	-1	3/4	0	3/2	3/2	-1/4	-1
-2	4	2	-6	-2	9	2	-3	-1/8	-7/8	9/8	0	9/8	9/8	-1/8	-1/2
-1	2	-1	-1	4	-1	-1	3	0	0	1/6	0	-1/6	-1/2	0	0
-1	3	-1	-4	1	1	8	-3	0	0	1/3	0	2/3	1	0	0
-1	2	-1	-3	4	3	1	-1	0	0	1/2	0	1/2	1/2	0	0
0	-1	-4	1	13	-1	-7	3	-1/2	1/2	1/3	0	1/6	1/2	0	0
0	0	-4	-2	10	1	2	-3	0	-1	2/3	0	-2/3	-1	0	0
0	-1	-4	-1	13	3	-5	-1	-1/2	-1/2	1	0	-1/2	-1/2	0	0
-2	3	2	1	-3	-1	1	3	0	0	1/4	0	0	0	-1/4	-1
-2	4	2	-2	-6	1	10	-3	0	0	1/2	0	0	0	1/2	2
-2	3	2	-1	-3	3	3	-1	0	0	3/4	0	0	0	1/4	1
-1	0	-1	3	6	-1	-5	3	-1/2	1/2	1/2	0	0	0	0	0
-1	1	-1	0	3	1	4	-3	0	-1	1	0	0	0	0	0
-1	0	-1	1	6	3	-3	-1	-1/2	-1/2	3/2	0	0	0	0	0
-2	1	2	7	-1	-5	-5	7	-1/2	1/2	1/4	0	0	-1	1/4	1
-2	2	2	4	-4	-3	4	1	0	0	1/2	0	0	1	-1/2	-2
-2	3	2	1	-7	-1	13	-5	0	-1	1	0	0	-1	1	-4
-2	1	2	5	-1	-1	-3	3	-1/2	1/2	3/4	0	0	0	-1/4	-1
-2	2	2	2	-4	1	6	-3	0	-1	3/2	0	0	0	1/2	2
-2	1	2	3	-1	3	-1	-1	-1/2	-1/2	9/4	0	0	0	1/4	1
-1	-1	-1	6	5	-3	-2	1	0	0	1	0	0	-1	0	0
0	-4	-4	8	14	-3	-8	1	-2	1	2	0	0	1	0	0
-2	0	2	8	-2	-3	0	1	0	0	3/2	0	0	0	-1/2	-2
-1	-3	-1	10	7	-3	-6	1	-2	1	3	0	0	0	0	0
-2	-2	2	14	0	-7	-6	5	-2	2	3/2	0	0	-3	1/2	2
-2	-1	2	11	-3	-5	3	-1	0	-1	3	0	0	3	-1	-4
-2	-2	2	12	0	-3	-4	1	-2	1	9/2	0	0	0	-1/2	-2

$r_1$	$r_2$	$r_3$	$r_4$	$r_6$	$r_8$	$r_{12}$	$r_{24}$	$b_1$	$b_2$	$b_3$	$b_4$	$b_5$	$b_6$	$b_7$	$b_8$
-2	-5	2	21	1	-9	-7	3	-8	7	9	0	0	-9	1	4
-2	3	2	0	1	-2	-4	6	-1/4	1/4	1/6	0	1/12	-1/4	0	0
-2	5	2	-6	-5	2	14	-6	0	-1	2/3	0	4/3	1	0	0
-2	3	2	-2	1	2	-2	2	-1/4	1/4	1/2	0	-1/4	-1/4	0	0
-2	4	2	-5	-2	4	7	-4	0	-1	1	0	1	1	0	0
-2	3	2	-4	1	6	0	-2	-1/4	-3/4	3/2	0	3/4	3/4	0	0
-2	0	2	7	2	-4	-5	4	-1	1	1	0	0	-1	0	0
-2	1	2	4	-1	-2	4	-2	0	-1	2	0	0	1	0	0
-2	-3	2	14	3	-6	-6	2	-4	3	6	0	0	-3	0	0
-3	5	5	-2	-5	-1	2	3	0	0	1/3	0	1/6	1/2	-1/2	-2
-3	6	5	-5	-8	1	11	-3	0	0	2/3	0	-2/3	-1	1	4
-3	5	5	-4	-5	3	4	-1	0	0	1	0	-1/2	-1/2	1/2	2
-2	2	2	0	4	-1	-4	3	-1/2	1/2	2/3	0	-1/6	-1/2	0	0
-2	3	2	-3	1	1	5	-3	0	-1	4/3	0	2/3	1	0	0
-2	2	2	-2	4	3	-2	-1	-1/2	-1/2	2	0	1/2	1/2	0	0
-3	3	5	2	-3	-1	-2	3	-1/2	1/2	1	0	0	0	-1/2	-2
-3	4	5	-1	-6	1	7	-3	0	-1	2	0	0	0	1	4
-3	3	5	0	-3	3	0	-1	-1/2	-1/2	3	0	0	0	1/2	2
-3	2	5	5	-4	-3	1	1	0	0	2	0	0	1	-1	-4
-2	-1	2	7	5	-3	-5	1	-2	1	4	0	0	-1	0	0
-3	0	5	9	-2	-3	-3	1	-2	1	6	0	0	0	-1	-4
-4	5	8	-1	-5	-1	-1	3	-1/2	1/2	4/3	0	1/6	1/2	-1	-4
-4	6	8	-4	-8	1	8	-3	0	-1	8/3	0	-2/3	-1	2	8
-4	5	8	-3	-5	3	1	-1	-1/2	-1/2	4	0	-1/2	-1/2	1	4
-4	2	8	6	-4	-3	-2	1	-2	1	8	0	0	1	-2	-8
-3	8	1	-4	-4	0	2	4	0	0	1/8	0	1/4	1/4	-3/8	-1
-3	9	1	-7	-7	2	11	-2	0	0	1/4	0	-1	-1	3/4	2
-3	10	1	-10	-10	4	20	-8	0	-1	1/2	0	4	4	-3/2	-4
-3	8	1	-6	-4	4	4	0	0	0	3/8	0	-3/4	-3/4	3/8	1
-3	9	1	-9	-7	6	13	-6	0	-1	3/4	0	3	3	-3/4	-2
-3	8	1	-8	-4	8	6	-4	0	-1	9/8	0	9/4	9/4	-3/8	-1
-2	5	-2	-2	5	0	-4	4	-1/4	1/4	1/4	0	-1/4	-1/4	1/4	0
-2	6	-2	-5	2	2	5	-2	0	0	1/2	0	1	1	-1/2	0
-2	5	-2	-4	5	4	-2	0	-1/4	1/4	3/4	0	3/4	3/4	-1/4	0
-1	3	-5	-3	11	2	-1	-2	0	-1	1	0	-1	-1	1	0
-3	6	1	2	-2	-4	-4	8	-1/4	1/4	1/8	0	0	-1	1/8	1
-3	7	1	-1	-5	-2	5	2	0	0	1/4	0	0	1	-1/4	-2
-3	8	1	-4	-8	0	14	-4	0	0	1/2	0	0	-1	1/2	4
-3	6	1	0	-2	0	-2	4	-1/4	1/4	3/8	0	0	0	-1/8	-1
-3	7	1	-3	-5	2	7	-2	0	0	3/4	0	0	0	1/4	2
-3	6	1	-2	-2	4	0	0	-1/4	1/4	9/8	0	0	0	1/8	1
-2	4	-2	1	4	-2	-1	2	0	0	1/2	0	0	-1	-1/2	0
-2	5	-2	-2	1	0	8	-4	0	-1	1	0	0	1	1	0
-2	4	-2	-1	4	2	1	-2	0	-1	3/2	0	0	0	1/2	0
-1	1	-5	3	13	-2	-7	2	-1	1	1	0	0	1	1	0
-3	5	1	3	-3	-2	1	2	0	0	3/4	0	0	0	-3/4	-2
-3	6	1	0	-6	0	10	-4	0	-1	3/2	0	0	0	3/2	4



$r_1$	$r_2$	$r_3$	$r_4$	$r_6$	$r_8$	$r_{12}$	$r_{24}$	$b_1$	$b_2$	$b_3$	$b_4$	$b_5$	$b_6$	$b_7$	$b_8$
-3	5	1	1	-3	2	3	-2	0	-1	9/4	0	0	0	3/4	2
-2	2	-2	5	6	-2	-5	2	-1	1	3/2	0	0	0	1/2	0
-3	3	1	9	-1	-6	-5	6	-1	1	3/4	0	0	-3	1/4	2
-3	4	1	6	-4	-4	4	0	0	0	3/2	0	0	3	-1/2	-4
-3	3	1	7	-1	-2	-3	2	-1	1	9/4	0	0	0	-1/4	-2
-2	1	-2	8	5	-4	-2	0	0	-1	3	0	0	-3	-1	0
-3	2	1	10	-2	-4	0	0	0	-1	9/2	0	0	0	-3/2	-4
-3	0	1	16	0	-8	-6	4	-4	4	9/2	0	0	-9	1/2	4
-3	7	1	-4	-1	1	2	1	0	0	1/2	0	-1/2	-1/2	0	0
-3	8	1	-7	-4	3	11	-5	0	-1	1	0	2	2	0	0
-3	7	1	-6	-1	5	4	-3	0	-1	3/2	0	3/2	3/2	0	0
-2	4	-2	-2	8	1	-4	1	-1/2	1/2	1	0	1/2	1/2	0	0
-3	5	1	2	1	-3	-4	5	-1/2	1/2	1/2	0	0	-1	0	0
-3	6	1	-1	-2	-1	5	-1	0	0	1	0	0	1	0	0
-3	5	1	0	1	1	-2	1	-1/2	1/2	3/2	0	0	0	0	0
-2	3	-2	1	7	-1	-1	-1	0	-1	2	0	0	-1	0	0
-3	4	1	3	0	-1	1	-1	0	-1	3	0	0	0	0	0
-3	2	1	9	2	-5	-5	3	-2	2	3	0	0	-3	0	0
-4	8	4	-3	-4	0	-1	4	-1/4	1/4	1/2	0	1/4	1/4	-1/2	-2
-4	9	4	-6	-7	2	8	-2	0	0	1	0	-1	1	1	4
-4	8	4	-5	-4	4	1	0	-1/4	1/4	3/2	0	-3/4	-3/4	1/2	2
-3	6	1	-4	2	2	2	-2	0	-1	2	0	1	1	0	0
-4	7	4	0	-5	-2	2	2	0	0	1	0	0	1	-1	-4
-4	8	4	-3	-8	0	11	-4	0	-1	2	0	0	-1	2	8
-4	7	4	-2	-5	2	4	-2	0	-1	3	0	0	0	1	4
-3	4	1	2	4	-2	-4	2	-1	1	2	0	0	-1	0	0
-4	5	4	4	-3	-2	-2	2	-1	1	3	0	0	0	-1	-4
-4	4	4	7	-4	-4	1	0	0	-1	6	0	0	3	-2	-8
-4	7	4	-3	-1	1	-1	1	-1/2	1/2	2	0	-1/2	-1/2	0	0
-4	6	4	0	-2	-1	2	-1	0	-1	4	0	0	1	0	0
-5	9	7	-5	-7	2	5	-2	0	-1	4	0	-1	-1	2	8
-5	7	7	1	-5	-2	-1	2	-1	1	4	0	0	1	-2	-8
-4	10	0	-3	0	-2	-3	6	-1/4	1/4	1/4	0	1/4	-3/4	-1/4	0
-4	12	0	-9	-6	2	15	-6	0	-1	1	0	4	3	-1	0
-4	10	0	-5	0	2	-1	2	-1/4	1/4	3/4	0	-3/4	-3/4	1/4	0
-4	11	0	-8	-3	4	8	-4	0	-1	3/2	0	3	3	-1/2	0
-4	10	0	-7	0	6	1	-2	-1/4	-3/4	9/4	0	9/4	9/4	-1/4	0
-4	7	0	4	1	-4	-4	4	-1	1	3/2	0	0	-3	-1/2	0
-4	8	0	1	-2	-2	5	-2	0	-1	3	0	0	3	1	0
-4	4	0	11	2	-6	-5	2	-4	3	9	0	0	-9	-1	0
-5	12	3	-5	-6	-1	3	3	0	0	1/2	0	1/2	3/2	-1	-4
-5	13	3	-8	-9	1	12	-3	0	0	1	0	-2	-3	2	8
-5	12	3	-7	-6	3	5	-1	0	0	3/2	0	-3/2	-3/2	1	4
-4	9	0	-3	3	-1	-3	3	-1/2	1/2	1	0	-1/2	-3/2	0	0
-4	10	0	-6	0	1	6	-3	0	-1	2	0	2	3	0	0
-4	9	0	-5	3	3	-1	-1	-1/2	-1/2	3	0	3/2	3/2	0	0
-5	10	3	-1	-4	-1	-1	3	-1/2	1/2	3/2	0	0	0	-1	-4
-5	11	3	-4	-7	1	8	-3	0	-1	3	0	0	0	2	8

$r_1$	$r_2$	$r_3$	$r_4$	$r_6$	$r_8$	$r_{12}$	$r_{24}$	$b_1$	$b_2$	$b_3$	$b_4$	$b_5$	$b_6$	$b_7$	$b_8$
-5	10	3	-3	-4	3	1	-1	-1/2	-1/2	9/2	0	0	0	1	4
-5	9	3	2	-5	-3	2	1	0	0	3	0	0	3	-2	-8
-4	6	0	4	4	-3	-4	1	-2	1	6	0	0	-3	0	0
-5	7	3	6	-3	-3	-2	1	-2	1	9	0	0	0	-2	-8
-6	12	6	-4	-6	-1	0	3	-1/2	1/2	2	0	1/2	3/2	-2	-8
-6	13	6	-7	-9	1	9	-3	0	-1	4	0	-2	-3	4	16
-6	12	6	-6	-6	3	2	-1	-1/2	-1/2	6	0	-3/2	-3/2	2	8
-6	9	6	3	-5	-3	-1	1	-2	1	12	0	0	3	-4	-16
-6	15	2	-6	-5	0	0	4	-1/4	1/4	3/4	0	3/4	3/4	-5/4	-4
-6	16	2	-9	-8	2	9	-2	0	0	3/2	0	-3	-3	5/2	8
-6	15	2	-8	-5	4	2	0	-1/4	1/4	9/4	0	-9/4	-9/4	5/4	4
-5	13	-1	-7	1	2	3	-2	0	-1	3	0	3	3	-1	0
-6	14	2	-3	-6	-2	3	2	0	0	3/2	0	0	3	-3/2	-8
-6	15	2	-6	-9	0	12	-4	0	-1	3	0	0	-3	3	16
-6	14	2	-5	-6	2	5	-2	0	-1	9/2	0	0	0	3/2	8
-5	11	-1	-1	3	-2	-3	2	-1	1	3	0	0	-3	-1	0
-6	12	2	1	-4	-2	-1	2	-1	1	9/2	0	0	0	-5/2	-8
-6	11	2	4	-5	-4	2	0	0	-1	9	0	0	9	-3	-16
-6	14	2	-6	-2	1	0	1	-1/2	1/2	3	0	-3/2	-3/2	0	0
-6	13	2	-3	-3	-1	3	-1	0	-1	6	0	0	3	0	0
-7	16	5	-8	-8	2	6	-2	0	-1	6	0	-3	-3	4	16
-7	14	5	-2	-6	-2	0	2	-1	1	6	0	0	3	-4	-16
-8	19	4	-7	-7	-1	1	3	-1/2	1/2	3	0	3/2	9/2	-4	-16
-8	20	4	-10	-10	1	10	-3	0	-1	6	0	-6	-9	8	32
-8	19	4	-9	-7	3	3	-1	-1/2	-1/2	9	0	-9/2	-9/2	4	16
-8	16	4	0	-6	-3	0	1	-2	1	18	0	0	9	-8	-32
-9	23	3	-11	-9	2	7	-2	0	-1	9	0	-9	-9	9	32
-9	21	3	-5	-7	-2	1	2	-1	1	9	0	0	9	-7	-32
2	-2	-2	2	4	-3	-6	9	1/8	-1/8	-1/24	1/6	1/24	-7/24	-1/8	0
2	-1	-2	-1	1	-1	3	3	0	0	-1/12	1/3	-1/6	1/6	1/4	0
2	0	-2	-4	-2	1	12	-3	0	0	-1/6	2/3	2/3	1/3	-1/2	0
2	1	-2	-7	-5	3	21	-9	0	-1	-1/3	4/3	-8/3	-7/3	1	0
2	-2	-2	0	4	1	-4	5	1/8	-1/8	-1/8	1/2	-1/8	-1/8	1/8	0
2	-1	-2	-3	1	3	5	-1	0	0	-1/4	1	1/2	1/2	-1/4	0
2	0	-2	-6	-2	5	14	-7	0	-1	-1/2	2	-2	-2	1/2	0
2	-2	-2	-2	4	5	-2	1	1/8	-1/8	-3/8	3/2	3/8	3/8	-1/8	0
2	-1	-2	-5	1	7	7	-5	0	-1	-3/4	3	-3/2	-3/2	1/4	0
2	-2	-2	-4	4	9	0	-3	1/8	-9/8	-9/8	9/2	-9/8	-9/8	1/8	0
3	-4	-5	1	10	-1	-3	3	0	0	1/3	-4/3	1/6	-1/6	-1/2	0
3	-3	-5	-2	7	1	6	-3	0	0	2/3	-8/3	-2/3	-1/3	1	0
3	-4	-5	-1	10	3	-1	-1	0	0	1	-4	-1/2	-1/2	1/2	0
4	-7	-8	3	19	-1	-9	3	1/2	-1/2	-4/3	16/3	-1/6	1/6	1	0
4	-6	-8	0	16	1	0	-3	0	-1	-8/3	32/3	2/3	1/3	-2	0
4	-7	-8	1	19	3	-7	-1	1/2	-3/2	-4	16	1/2	1/2	-1	0
2	-3	-2	3	3	-1	-1	3	0	0	1/4	-1	0	0	-1/4	0
2	-2	-2	0	0	1	8	-3	0	0	1/2	-2	0	0	1/2	0
2	-3	-2	1	3	3	1	-1	0	0	3/4	-3	0	0	1/4	0

$r_1$	$r_2$	$r_3$	$r_4$	$r_6$	$r_8$	$r_{12}$	$r_{24}$	$b_1$	$b_2$	$b_3$	$b_4$	$b_5$	$b_6$	$b_7$	$b_8$
3	-6	-5	5	12	-1	-7	3	1/2	-1/2	-1	4	0	0	1/2	0
3	-5	-5	2	9	1	2	-3	0	-1	-2	8	0	0	-1	0
3	-6	-5	3	12	3	-5	-1	1/2	-3/2	-3	12	0	0	-1/2	0
2	-5	-2	9	5	-5	-7	7	1/2	-1/2	-1/4	1	0	-1	-1/4	0
2	-4	-2	6	2	-3	2	1	0	0	-1/2	2	0	1	1/2	0
2	-3	-2	3	-1	-1	11	-5	0	-1	-1	4	0	-1	-1	0
2	-5	-2	7	5	-1	-5	3	1/2	-1/2	-3/4	3	0	0	1/4	0
2	-4	-2	4	2	1	4	-3	0	-1	-3/2	6	0	0	-1/2	0
2	-5	-2	5	5	3	-3	-1	1/2	-3/2	-9/4	9	0	0	-1/4	0
3	-7	-5	8	11	-3	-4	1	0	0	2	-8	0	-1	-1	0
4	-10	-8	10	20	-3	-10	1	2	-3	-8	32	0	1	2	0
2	-6	-2	10	4	-3	-2	1	0	0	3/2	-6	0	0	-1/2	0
3	-9	-5	12	13	-3	-8	1	2	-3	-6	24	0	0	1	0
2	-8	-2	16	6	-7	-8	5	2	-2	-3/2	6	0	-3	-1/2	0
2	-7	-2	13	3	-5	1	-1	0	-1	-3	12	0	3	1	0
2	-8	-2	14	6	-3	-6	1	2	-3	-9/2	18	0	0	1/2	0
2	-11	-2	23	7	-9	-9	3	8	-9	-9	36	0	-9	-1	0
1	0	1	0	-2	-2	0	6	0	0	1/24	-1/6	1/12	5/12	-1/8	-1/2
1	1	1	-3	-5	0	9	0	0	0	1/12	-1/3	-1/3	-2/3	1/4	1
1	2	1	-6	-8	2	18	-6	0	0	1/6	-2/3	4/3	5/3	-1/2	-2
1	0	1	-2	-2	2	2	2	0	0	1/8	-1/2	-1/4	-1/4	1/8	1/2
1	1	1	-5	-5	4	11	-4	0	0	1/4	-1	1	1	-1/4	-1
1	0	1	-4	-2	6	4	-2	0	0	3/8	-3/2	3/4	3/4	-1/8	-1/2
2	-3	-2	2	7	-2	-6	6	1/4	-1/4	-1/6	2/3	-1/12	-5/12	0	0
2	-2	-2	-1	4	0	3	0	0	0	-1/3	4/3	1/3	2/3	0	0
2	-1	-2	-4	1	2	12	-6	0	-1	-2/3	8/3	-4/3	-5/3	0	0
2	-3	-2	0	7	2	-4	2	1/4	-1/4	-1/2	2	1/4	1/4	0	0
2	-2	-2	-3	4	4	5	-4	0	-1	-1	4	-1	-1	0	0
2	-3	-2	-2	7	6	-2	-2	1/4	-5/4	-3/2	6	-3/4	-3/4	0	0
3	-5	-5	1	13	0	-3	0	0	0	4/3	-16/3	-1/3	-2/3	0	0
4	-8	-8	3	22	0	-9	0	1	-2	-16/3	64/3	1/3	2/3	0	0
1	-2	1	4	0	-2	-4	6	1/4	-1/4	-1/8	1/2	0	0	-1/8	-1/2
1	-1	1	1	-3	0	5	0	0	0	-1/4	1	0	0	1/4	1
1	0	1	-2	-6	2	14	-6	0	-1	-1/2	2	0	0	-1/2	-2
1	-2	1	2	0	2	-2	2	1/4	-1/4	-3/8	3/2	0	0	1/8	1/2
1	-1	1	-1	-3	4	7	-4	0	-1	-3/4	3	0	0	-1/4	-1
1	-2	1	0	0	6	0	-2	1/4	-5/4	-9/8	9/2	0	0	-1/8	-1/2
2	-4	-2	3	6	0	-1	0	0	0	1	-4	0	0	0	0
3	-7	-5	5	15	0	-7	0	1	-2	-4	16	0	0	0	0
1	-3	1	7	-1	-4	-1	4	0	0	1/4	-1	0	1	-1/4	-1
1	-2	1	4	-4	-2	8	-2	0	0	1/2	-2	0	-1	1/2	2
1	-3	1	5	-1	0	1	0	0	0	3/4	-3	0	0	1/4	1
2	-6	-2	9	8	-4	-7	4	1	-1	-1	4	0	-1	0	0
2	-5	-2	6	5	-2	2	-2	0	-1	-2	8	0	1	0	0
2	-6	-2	7	8	0	-5	0	1	-2	-3	12	0	0	0	0
1	-5	1	11	1	-4	-5	4	1	-1	-3/4	3	0	0	-1/4	-1
1	-4	1	8	-2	-2	4	-2	0	-1	-3/2	6	0	0	1/2	2

$r_1$	$r_2$	$r_3$	$r_4$	$r_6$	$r_8$	$r_{12}$	$r_{24}$	$b_1$	$b_2$	$b_3$	$b_4$	$b_5$	$b_6$	$b_7$	$b_8$
1	-5	1	9	1	0	-3	0	1	-2	-9/4	9	0	0	1/4	1
1	-6	1	14	0	-6	-2	2	0	0	3/2	-6	0	3	-1/2	-2
2	-9	-2	16	9	-6	-8	2	4	-5	-6	24	0	-3	0	0
1	-8	1	18	2	-6	-6	2	4	-5	-9/2	18	0	0	-1/2	-2
1	-1	1	0	1	-1	0	3	0	0	1/6	-2/3	-1/6	1/6	0	0
1	0	1	-3	-2	1	9	-3	0	0	1/3	-4/3	2/3	1/3	0	0
1	-1	1	-2	1	3	2	-1	0	0	1/2	-2	1/2	1/2	0	0
2	-4	-2	2	10	-1	-6	3	1/2	-1/2	-2/3	8/3	1/6	-1/6	0	0
2	-3	-2	-1	7	1	3	-3	0	-1	-4/3	16/3	-2/3	-1/3	0	0
2	-4	-2	0	10	3	-4	-1	1/2	-3/2	-2	8	-1/2	-1/2	0	0
1	-3	1	4	3	-1	-4	3	1/2	-1/2	-1/2	2	0	0	0	0
1	-2	1	1	0	1	5	-3	0	-1	-1	4	0	0	0	0
1	-3	1	2	3	3	-2	-1	1/2	-3/2	-3/2	6	0	0	0	0
1	-4	1	7	2	-3	-1	1	0	0	1	-4	0	1	0	0
2	-7	-2	9	11	-3	-7	1	2	-3	-4	16	0	-1	0	0
1	-6	1	11	4	-3	-5	1	2	-3	-3	12	0	0	0	0
0	0	4	1	-2	-2	-3	6	1/4	-1/4	-1/12	1/3	1/12	5/12	-1/4	-1
0	1	4	-2	-5	0	6	0	0	0	-1/6	2/3	-1/3	-2/3	1/2	2
0	2	4	-5	-8	2	15	-6	0	-1	-1/3	4/3	4/3	5/3	-1	-4
0	0	4	-1	-2	2	-1	2	1/4	-1/4	-1/4	1	-1/4	-1/4	1/4	1
0	1	4	-4	-5	4	8	-4	0	-1	-1/2	2	1	1	-1/2	-2
0	0	4	-3	-2	6	1	-2	1/4	-5/4	-3/4	3	3/4	3/4	-1/4	-1
1	-2	1	0	4	0	0	0	0	0	2/3	-8/3	1/3	2/3	0	0
2	-5	-2	2	13	0	-6	0	1	-2	-8/3	32/3	-1/3	-2/3	0	0
0	-1	4	2	-3	0	2	0	0	0	1/2	-2	0	0	1/2	2
1	-4	1	4	6	0	-4	0	1	-2	-2	8	0	0	0	0
0	-3	4	8	-1	-4	-4	4	1	-1	-1/2	2	0	1	-1/2	-2
0	-2	4	5	-4	-2	5	-2	0	-1	-1	4	0	-1	1	4
0	-3	4	6	-1	0	-2	0	1	-2	-3/2	6	0	0	1/2	2
0	-6	4	15	0	-6	-5	2	4	-5	-3	12	0	3	-1	-4
0	-1	4	1	1	-1	-3	3	1/2	-1/2	-1/3	4/3	-1/6	1/6	0	0
0	0	4	-2	-2	1	6	-3	0	-1	-2/3	8/3	2/3	1/3	0	0
0	-1	4	-1	1	3	-1	-1	1/2	-3/2	-1	4	1/2	1/2	0	0
0	-4	4	8	2	-3	-4	1	2	-3	-2	8	0	1	0	0
-1	1	7	-1	-5	0	3	0	0	0	1/3	-4/3	-1/3	-2/3	1	4
0	-2	4	1	4	0	-3	0	1	-2	-4/3	16/3	1/3	2/3	0	0
-1	-1	7	3	-3	0	-1	0	1	-2	-1	4	0	0	1	4
-2	1	10	0	-5	0	0	0	1	-2	-2/3	8/3	-1/3	-2/3	2	8
0	3	0	-1	-1	-1	-3	7	1/8	-1/8	0	0	1/8	1/8	-1/4	-1/2
0	4	0	-4	-4	1	6	1	0	0	0	0	-1/2	-1/2	1/2	1
0	5	0	-7	-7	3	15	-5	0	0	0	0	2	2	-1	-2
0	3	0	-3	-1	3	-1	3	1/8	-1/8	0	0	-3/8	-3/8	1/4	1/2
0	4	0	-6	-4	5	8	-3	0	0	0	0	3/2	3/2	-1/2	-1
0	3	0	-5	-1	7	1	-1	1/8	-1/8	0	0	9/8	9/8	-1/4	-1/2
1	1	-3	-2	5	1	0	1	0	0	0	0	1/2	1/2	-1/2	0
1	2	-3	-5	2	3	9	-5	0	-1	0	0	-2	-2	1	0
1	1	-3	-4	5	5	2	-3	0	-1	0	0	-3/2	-3/2	1/2	0

$r_1$	$r_2$	$r_3$	$r_4$	$r_6$	$r_8$	$r_{12}$	$r_{24}$	$b_1$	$b_2$	$b_3$	$b_4$	$b_5$	$b_6$	$b_7$	$b_8$
2	-2	-6	0	14	1	-6	1	1/2	-1/2	0	0	-1/2	-1/2	1	0
1	-1	-3	4	7	-3	-6	5	1/2	-1/2	0	0	0	-1	-1/2	0
1	0	-3	1	4	-1	3	-1	0	0	0	0	0	1	1	0
1	-1	-3	2	7	1	-4	1	1/2	-1/2	0	0	0	0	1/2	0
2	-3	-6	3	13	-1	-3	-1	0	-1	0	0	0	-1	-2	0
0	0	0	6	0	-3	-4	5	1/2	-1/2	0	0	0	0	-1/2	-1
0	1	0	3	-3	-1	5	-1	0	0	0	0	0	0	1	-2
0	0	0	4	0	1	-2	1	1/2	-1/2	0	0	0	0	1/2	1
1	-2	-3	5	6	-1	-1	-1	0	-1	0	0	0	0	-1	0
1	-4	-3	11	8	-5	-7	3	2	-2	0	0	0	-3	-1	0
0	-3	0	13	1	-5	-5	3	2	-2	0	0	0	0	-1	-2
0	2	0	-1	2	0	-3	4	1/4	-1/4	0	0	-1/4	-1/4	0	0
0	3	0	-4	-1	2	6	-2	0	0	0	0	1	1	0	0
0	2	0	-3	2	4	-1	0	1/4	-1/4	0	0	3/4	3/4	0	0
1	0	-3	-2	8	2	0	-2	0	-1	0	0	-1	-1	0	0
1	-2	-3	4	10	-2	-6	2	1	-1	0	0	0	-1	0	0
0	-1	0	6	3	-2	-4	2	1	-1	0	0	0	0	0	0
-1	4	3	-3	-4	1	3	1	0	0	0	0	-1/2	-1/2	1/2	2
-1	5	3	-6	-7	3	12	-5	0	-1	0	0	2	2	-1	-4
-1	4	3	-5	-4	5	5	-3	0	-1	0	0	3/2	3/2	-1/2	-2
0	1	0	-1	5	1	-3	1	1/2	-1/2	0	0	1/2	1/2	0	0
-1	2	3	3	-2	-3	-3	5	1/2	-1/2	0	0	0	1	-1/2	-2
-1	3	3	0	-5	-1	6	-1	0	0	0	0	0	-1	1	4
-1	2	3	1	-2	1	-1	1	1/2	-1/2	0	0	0	0	1/2	2
-1	1	3	4	-3	-1	2	-1	0	-1	0	0	0	0	1	4
-1	-1	3	10	-1	-5	-4	3	2	-2	0	0	0	3	-1	-4
-1	3	3	-3	-1	2	3	-2	0	-1	0	0	1	1	0	0
-1	1	3	3	1	-2	-3	2	1	-1	0	0	0	1	0	0
-2	4	6	-2	-4	1	0	1	1/2	-1/2	0	0	-1/2	-1/2	1	4
-2	3	6	1	-5	-1	3	-1	0	-1	0	0	0	-1	2	8
-1	6	-1	-3	0	-1	1	3	0	0	0	0	-1/2	1/2	1/2	0
-1	7	-1	-6	-3	1	10	-3	0	0	0	0	2	1	-1	0
-1	6	-1	-5	0	3	3	-1	0	0	0	0	3/2	3/2	-1/2	0
0	3	-4	-1	9	-1	-5	3	1/2	-1/2	0	0	1/2	-1/2	-1	0
0	4	-4	-4	6	1	4	-3	0	-1	0	0	-2	-1	2	0
0	3	-4	-3	9	3	-3	-1	1/2	-3/2	0	0	-3/2	-3/2	1	0
-1	4	-1	1	2	-1	-3	3	1/2	-1/2	0	0	0	0	-1/2	0
-1	5	-1	-2	-1	1	6	-3	0	-1	0	0	0	0	1	0
-1	4	-1	-1	2	3	-1	-1	1/2	-3/2	0	0	0	0	1/2	0
-1	3	-1	4	1	-3	0	1	0	0	0	0	0	3	1	0
0	0	-4	6	10	-3	-6	1	2	-3	0	0	0	-3	-2	0
-1	1	-1	8	3	-3	-4	1	2	-3	0	0	0	0	-1	0
-2	7	2	-2	-3	-2	-2	6	1/4	-1/4	0	0	1/4	5/4	-1/2	-2
-2	8	2	-5	-6	0	7	0	0	0	0	0	-1	-2	1	4
-2	9	2	-8	-9	2	16	-6	0	-1	0	0	4	5	-2	-8
-2	7	2	-4	-3	2	0	2	1/4	-1/4	0	0	-3/4	-3/4	1/2	2
-2	8	2	-7	-6	4	9	-4	0	-1	0	0	3	3	-1	-4
-2	7	2	-6	-3	6	2	-2	1/4	-5/4	0	0	9/4	9/4	-1/2	-2
-1	5	-1	-3	3	0	1	0	0	0	0	0	1	2	0	0

$r_1$	$r_2$	$r_3$	$r_4$	$r_6$	$r_8$	$r_{12}$	$r_{24}$	$b_1$	$b_2$	$b_3$	$b_4$	$b_5$	$b_6$	$b_7$	$b_8$
0	2	-4	-1	12	0	-5	0	1	-2	0	0	-1	-2	0	0
-2	6	2	-1	-4	0	3	0	0	0	0	0	0	0	1	4
-1	3	-1	1	5	0	-3	0	1	-2	0	0	0	0	0	0
-2	4	2	5	-2	-4	-3	4	1	-1	0	0	0	3	-1	-4
-2	5	2	2	-5	-2	6	-2	0	-1	0	0	0	-3	2	8
-2	4	2	3	-2	0	-1	0	1	-2	0	0	0	0	1	4
-2	1	2	12	-1	-6	-4	2	4	-5	0	0	0	9	-2	-8
-2	6	2	-2	0	-1	-2	3	1/2	-1/2	0	0	-1/2	1/2	0	0
-2	7	2	-5	-3	1	7	-3	0	-1	0	0	1/2	1	0	0
-2	6	2	-4	0	3	0	-1	1/2	-3/2	0	0	3/2	3/2	0	0
-2	3	2	5	1	-3	-3	1	2	-3	0	0	0	3/3	0	0
-3	8	5	-4	-6	0	4	0	0	0	0	0	-1	-2	2	8
-2	5	2	-2	3	0	-2	0	1	-2	0	0	1	2	0	0
-3	6	5	0	-4	0	0	0	1	-2	0	0	0	0	2	8
-4	8	8	-3	-6	0	1	0	1	-2	0	0	-1	-2	4	16
-3	11	1	-6	-5	1	4	1	0	0	0	0	-3/2	-3/2	3/2	4
-3	12	1	-9	-8	3	13	-5	0	-1	0	0	6	6	-3	-8
-3	11	1	-8	-5	5	6	-3	0	-1	0	0	9/2	9/2	-3/2	-4
-2	8	-2	-4	4	1	-2	1	1/2	-1/2	0	0	3/2	3/2	-1	0
-3	9	1	0	-3	-3	-2	5	1/2	-1/2	0	0	0	3	-1/2	-4
-3	10	1	-3	-6	-1	7	-1	0	0	0	0	0	-3	1	8
-3	9	1	-2	-3	1	0	1	1/2	-1/2	0	0	0	0	1/2	4
-2	7	-2	-1	3	-1	1	-1	0	-1	0	0	0	3	2	0
-3	8	1	1	-4	-1	3	-1	0	-1	0	0	0	0	3	8
-3	6	1	7	-2	-5	-3	3	2	-2	0	0	0	9	-1	-8
-3	10	1	-6	-2	2	4	-2	0	-1	0	0	3	3	0	0
-3	8	1	0	0	-2	-2	2	1	-1	0	0	0	3	0	0
-4	11	4	-5	-5	1	1	1	1/2	-1/2	0	0	-3/2	-3/2	2	8
-4	10	4	-2	-6	-1	4	-1	0	-1	0	0	0	-3	4	16
-4	13	0	-5	-1	-1	-1	3	1/2	-1/2	0	0	-3/2	3/2	1	0
-4	14	0	-8	-4	1	8	-3	0	-1	0	0	6	3	-2	0
-4	13	0	-7	-1	3	1	-1	1/2	-3/2	0	0	9/2	9/2	-1	0
-4	10	0	2	0	-3	-2	1	2	-3	0	0	0	9	2	0
-5	15	3	-7	-7	0	5	0	0	0	0	0	-3	-6	4	16
-4	12	0	-5	2	0	-1	0	1	-2	0	0	3	6	0	0
-5	13	3	-3	-5	0	1	0	1	-2	0	0	0	0	4	16
-6	15	6	-6	-7	0	2	0	1	-2	0	0	-3	-6	8	32
-6	18	2	-8	-6	1	2	1	1/2	-1/2	0	0	-9/2	-9/2	5	16
-6	17	2	-5	-7	-1	5	-1	0	-1	0	0	0	-9	6	32
-8	22	4	-9	-8	0	3	0	1	-2	0	0	-9	-18	16	64
3	-1	-1	-1	-1	0	1	4	0	0	-1/8	1/2	-1/4	-1/4	3/8	1/2
3	0	-1	-4	-4	2	10	-2	0	0	-1/4	1	1	1	-3/4	-1
3	1	-1	-7	-7	4	19	-8	0	-1	-1/2	2	-4	-4	3/2	2
3	-1	-1	-3	-1	4	3	0	0	0	-3/8	3/2	3/4	3/4	-3/8	-1/2
3	0	-1	-6	-4	6	12	-6	0	-1	-3/4	3	-3	-3	3/4	1
3	-1	-1	-5	-1	8	5	-4	0	-1	-9/8	9/2	-9/4	-9/4	3/8	1/2
4	-4	-4	1	8	0	-5	4	-1/4	1/4	1/2	-2	1/4	1/4	-1/2	0
4	-3	-4	-2	5	2	4	-2	0	0	1	-4	-1	-1	1	0
4	-4	-4	-1	8	4	-3	0	-1/4	1/4	3/2	-6	-3/4	-3/4	1/2	0

$r_1$	$r_2$	$r_3$	$r_4$	$r_6$	$r_8$	$r_{12}$	$r_{24}$	$b_1$	$b_2$	$b_3$	$b_4$	$b_5$	$b_6$	$b_7$	$b_8$
5	-6	-7	0	14	2	-2	-2	0	-1	-4	16	1	1	-2	0
3	-3	-1	5	1	-4	-5	8	-1/4	1/4	1/8	-1/2	0	1	1/8	-1/2
3	-2	-1	2	-2	-2	4	2	0	0	1/4	-1	0	-1	-1/4	1
3	-1	-1	-1	-5	0	13	-4	0	0	1/2	-2	0	1	1/2	-2
3	-3	-1	3	1	0	-3	4	-1/4	1/4	3/8	-3/2	0	0	-1/8	1/2
3	-2	-1	0	-2	2	6	-2	0	0	3/4	-3	0	0	1/4	-1
3	-3	-1	1	1	4	-1	0	-1/4	1/4	9/8	-9/2	0	0	1/8	-1/2
4	-5	-4	4	7	-2	-2	2	0	0	-1	4	0	1	1	0
4	-4	-4	1	4	0	7	-4	0	-1	-2	8	0	-1	-2	0
4	-5	-4	2	7	2	0	-2	0	-1	-3	12	0	0	-1	0
5	-8	-7	6	16	-2	-8	2	-1	1	4	-16	0	-1	-2	0
3	-4	-1	6	0	-2	0	2	0	0	-3/4	3	0	0	3/4	1
3	-3	-1	3	-3	0	9	-4	0	-1	-3/2	6	0	0	-3/2	-2
3	-4	-1	4	0	2	2	-2	0	-1	-9/4	9	0	0	-3/4	-1
4	-7	-4	8	9	-2	-6	2	-1	1	3	-12	0	0	-1	0
3	-6	-1	12	2	-6	-6	6	-1	1	3/4	-3	0	3	1/4	-1
3	-5	-1	9	-1	-4	3	0	0	0	3/2	-6	0	-3	-1/2	2
3	-6	-1	10	2	-2	-4	2	-1	1	9/4	-9	0	0	-1/4	1
4	-8	-4	11	8	-4	-3	0	0	-1	-6	24	0	3	2	0
3	-7	-1	13	1	-4	-1	0	0	-1	-9/2	18	0	0	3/2	2
3	-9	-1	19	3	-8	-7	4	-4	4	9/2	-18	0	9	1/2	-2
3	-2	-1	-1	2	1	1	1	0	0	-1/2	2	1/2	1/2	0	0
3	-1	-1	-4	-1	3	10	-5	0	-1	-1	4	-2	-2	0	0
3	-2	-1	-3	2	5	3	-3	0	-1	-3/2	6	-3/2	-3/2	0	0
4	-5	-4	1	11	1	-5	1	-1/2	1/2	2	-8	-1/2	-1/2	0	0
3	-4	-1	5	4	-3	-5	5	-1/2	1/2	1/2	-2	0	1	0	0
3	-3	-1	2	1	-1	4	-1	0	0	1	-4	0	-1	0	0
3	-4	-1	3	4	1	-3	1	-1/2	1/2	3/2	-6	0	0	0	0
4	-6	-4	4	10	-1	-2	-1	0	-1	-4	16	0	1	0	0
3	-5	-1	6	3	-1	0	-1	0	-1	-3	12	0	0	0	0
3	-7	-1	12	5	-5	-6	3	-2	2	3	-12	0	3	0	0
2	-1	2	0	-1	0	-2	4	-1/4	1/4	1/4	-1	-1/4	-1/4	1/4	1
2	0	2	-3	-4	2	7	-2	0	0	1/2	-2	1	1	-1/2	-2
2	-1	2	-2	-1	4	0	0	-1/4	1/4	3/4	-3	3/4	3/4	-1/4	-1
3	-3	-1	-1	5	2	1	-2	0	-1	-2	8	-1	-1	0	0
2	-2	2	3	-2	-2	1	2	0	0	-1/2	2	0	-1	1/2	2
2	-1	2	0	-5	0	10	-4	0	-1	-1	4	0	1	-1	-4
2	-2	2	1	-2	2	3	-2	0	-1	-3/2	6	0	0	-1/2	-2
3	-5	-1	5	7	-2	-5	2	-1	1	2	-8	0	1	0	0
2	-4	2	7	0	-2	-3	2	-1	1	3/2	-6	0	0	1/2	2
2	-5	2	10	-1	-4	0	0	0	-1	-3	12	0	-3	1	4
2	-2	2	0	2	1	-2	1	-1/2	1/2	1	-4	1/2	1/2	0	0
2	-3	2	3	1	-1	1	-1	0	-1	-2	8	0	-1	0	0
1	0	5	-2	-4	2	4	-2	0	-1	-1	4	1	1	-1	-4
1	-2	5	4	-2	-2	-2	2	-1	1	1	-4	0	-1	1	4
2	1	-2	0	3	-2	-4	6	-1/4	1/4	0	0	-1/4	3/4	1/2	0
2	3	-2	-6	-3	2	14	-6	0	-1	0	0	-4	-3	2	0
2	1	-2	-2	3	2	-2	2	-1/4	1/4	0	0	3/4	3/4	-1/2	0

$r_1$	$r_2$	$r_3$	$r_4$	$r_6$	$r_8$	$r_{12}$	$r_{24}$	$b_1$	$b_2$	$b_3$	$b_4$	$b_5$	$b_6$	$b_7$	$b_8$
2	2	-2	-5	0	4	7	-4	0	-1	0	0	-3	-3	1	0
2	1	-2	-4	3	6	0	-2	-1/4	-3/4	0	0	-9/4	-9/4	1/2	0
2	-2	-2	7	4	-4	-5	4	-1	1	0	0	0	3	1	0
2	-1	-2	4	1	-2	4	-2	0	-1	0	0	0	-3	-2	0
2	-5	-2	14	5	-6	-6	2	-4	3	0	0	0	9	2	0
1	3	1	-2	-3	-1	2	3	0	0	0	0	-1/2	-3/2	1/2	2
1	4	1	-5	-6	1	11	-3	0	0	0	0	2	3	-1	-4
1	3	1	-4	-3	3	4	-1	0	0	0	0	3/2	3/2	-1/2	-2
2	0	-2	0	6	-1	-4	3	-1/2	1/2	0	0	1/2	3/2	0	0
2	1	-2	-3	3	1	5	-3	0	-1	0	0	-2	-3	0	0
2	0	-2	-2	6	3	-2	-1	-1/2	-1/2	0	0	-3/2	-3/2	0	0
1	1	1	2	-1	-1	-2	3	-1/2	1/2	0	0	0	0	1/2	2
1	2	1	-1	-4	1	7	-3	0	-1	0	0	0	0	-1	-4
1	1	1	0	-1	3	0	-1	-1/2	-1/2	0	0	0	0	-1/2	-2
1	0	1	5	-2	-3	1	1	0	0	0	0	0	-3	1	4
2	-3	-2	7	-3	-5	1	0	-2	1	0	0	0	3	0	0
1	-2	1	9	0	-3	-3	1	-2	1	0	0	0	0	1	4
0	3	4	-1	-3	-1	-1	3	-1/2	1/2	0	0	-1/2	-3/2	1	4
0	4	4	-4	-6	1	8	-3	0	-1	0	0	2	3	-2	-8
0	3	4	-3	-3	3	1	-1	-1/2	-1/2	0	0	3/2	3/2	-1	-4
0	0	4	6	-2	-3	-2	1	-2	1	0	0	0	-3	2	8
0	6	0	-3	-2	0	-1	4	-1/4	1/4	0	0	-3/4	-3/4	1	2
0	7	0	-6	-5	2	8	-2	0	0	0	0	3	3	-2	-4
0	6	0	-5	-2	4	1	0	-1/4	1/4	0	0	9/4	9/4	-1	-2
1	4	-3	-4	4	2	2	-2	0	-1	0	0	-3	-3	2	0
1	2	-3	2	6	-2	-4	2	-1	1	0	0	0	3	2	0
0	3	0	4	-1	-2	-2	2	-1	1	0	0	0	0	2	4
0	5	0	-3	1	1	-1	1	-1/2	1/2	0	0	3/2	3/2	0	0
-1	7	3	-5	-5	2	5	-2	0	-1	0	0	3	3	-2	-8
-1	5	3	1	-3	-2	-1	2	-1	1	0	0	0	-3	2	8
-2	10	2	-4	-4	-1	0	3	-1/2	1/2	0	0	-3/2	-9/2	2	8
-2	11	2	-7	-7	1	9	-3	0	-1	0	0	6	9	-4	-16
-2	10	2	-6	-4	3	2	-1	-1/2	-1/2	0	0	9/2	9/2	-2	-8
-2	7	2	3	-3	-3	-1	1	-2	1	0	0	0	-9	4	16
-3	14	1	-8	-6	2	6	-2	0	-1	0	0	9	9	-6	-16
-3	12	1	-2	-4	-2	0	2	-1	1	0	0	0	-9	2	16
5	-3	-3	0	3	-1	0	3	0	0	1/2	-2	1/2	-1/2	-1	0
5	-2	-3	-3	0	1	9	-3	0	0	1	-4	-2	-1	2	0
5	-3	-3	-2	3	3	2	-1	0	0	3/2	-6	-3/2	-3/2	1	0
6	-6	-6	2	12	-1	-6	3	1/2	-1/2	-2	8	-1/2	1/2	2	0
6	-5	-6	-1	9	1	3	-3	0	-1	-4	16	2	1	-4	0
6	-6	-6	0	12	3	-4	-1	1/2	-3/2	-6	24	3/2	3/2	-2	0
5	-5	-3	4	5	-1	-4	3	1/2	-1/2	-3/2	6	0	0	1	0
5	-4	-3	1	2	1	5	-3	0	-1	-3	12	0	0	-2	0
5	-5	-3	2	5	3	-2	-1	1/2	-3/2	-9/2	18	0	0	-1	0
5	-6	-3	7	4	-3	-1	1	0	0	3	-12	0	-3	-2	0
6	-9	-6	9	13	-3	-7	1	2	-3	-12	48	0	3	4	0
5	-8	-3	11	6	-3	-5	1	2	-3	-9	36	0	0	2	0
4	-2	0	1	0	-2	-3	6	1/4	-1/4	-1/4	1	-1/4	-5/4	1/4	1



$r_1$	$r_2$	$r_3$	$r_4$	$r_6$	$r_8$	$r_{12}$	$r_{24}$	$b_1$	$b_2$	$b_3$	$b_4$	$b_5$	$b_6$	$b_7$	$b_8$
4	-1	0	-2	-3	0	6	0	0	0	-1/2	2	1	2	-1/2	-2
4	0	0	-5	-6	2	15	-6	0	-1	-1	4	-4	-5	1	4
4	-2	0	-1	0	2	-1	2	1/4	-1/4	-3/4	3	3/4	3/4	-1/4	-1
4	-1	0	-4	-3	4	8	-4	0	-1	-3/2	6	-3	-3	1/2	2
4	-2	0	-3	0	6	1	-2	1/4	-5/4	-9/4	9	-9/4	-9/4	1/4	1
5	-4	-3	0	6	0	0	0	0	0	2	-8	-1	-2	0	0
6	-7	-6	2	15	0	-6	0	1	-2	-8	32	1	2	0	0
4	-3	0	2	-1	0	2	0	0	0	3/2	-6	0	0	-1/2	-2
5	-6	-3	4	8	0	-4	0	1	-2	-6	24	0	0	0	0
4	-5	0	8	1	-4	-4	4	1	-1	-3/2	6	0	-3	1/2	2
4	-4	0	5	-2	-2	5	-2	0	-1	-3	12	0	3	-1	-4
4	-5	0	6	1	0	-2	0	1	-2	-9/2	18	0	0	-1/2	-2
4	-8	0	15	2	-6	-5	2	4	-5	-9	36	0	-9	1	4
4	-3	0	1	3	-1	-3	3	1/2	-1/2	-1	4	1/2	-1/2	0	0
4	-2	0	-2	0	1	6	-3	0	-1	-2	8	-2	-1	0	0
4	-3	0	-1	3	3	-1	-1	1/2	-3/2	-3	12	-3/2	-3/2	0	0
4	-6	0	8	4	-3	-4	1	2	-3	-6	24	0	-3	0	0
3	-1	3	-1	-3	0	3	0	0	0	1	-4	1	2	-1	-4
4	-4	0	1	6	0	-3	0	1	-2	-4	16	-1	-2	0	0
3	-3	3	3	-1	0	-1	0	1	-2	-3	12	0	0	-1	-4
2	-1	6	0	-3	0	0	0	1	-2	-2	8	1	2	-2	-8
3	2	-1	-3	-2	1	3	1	0	0	0	0	3/2	3/2	-3/2	-2
3	3	-1	-6	-5	3	12	-5	0	-1	0	0	-6	-6	3	4
3	2	-1	-5	-2	5	5	-3	0	-1	0	0	-9/2	-9/2	3/2	2
4	-1	-4	-1	7	1	-3	1	1/2	-1/2	0	0	-3/2	-3/2	2	0
3	0	-1	3	0	-3	-3	5	1/2	-1/2	0	0	0	-3	-1/2	2
3	1	-1	0	-3	-1	6	-1	0	0	0	0	0	3	1	-4
3	0	-1	1	0	1	-1	1	1/2	-1/2	0	0	0	0	1/2	-2
4	-2	-4	2	6	-1	0	-1	0	-1	0	0	0	-3	-4	0
3	-1	-1	4	-1	-1	2	-1	0	-1	0	0	0	0	-3	-4
3	-3	-1	10	1	-5	-4	3	2	-2	0	0	0	-9	-1	4
3	1	-1	-3	1	2	3	-2	0	-1	0	0	-3	-3	0	0
3	-1	-1	3	3	-2	-3	2	1	-1	0	0	0	-3	0	0
2	2	2	-2	-2	1	0	1	1/2	-1/2	0	0	3/2	3/2	-1	-4
2	1	2	1	-3	-1	3	-1	0	-1	0	0	0	3	-2	-8
2	4	-2	-2	2	-1	-2	3	1/2	-1/2	0	0	3/2	-3/2	-2	0
2	5	-2	-5	-1	1	7	-3	0	-1	0	0	-6	-3	4	0
2	4	-2	-4	2	3	0	-1	1/2	-3/2	0	0	-9/2	-9/2	2	0
2	1	-2	5	3	-3	-3	1	2	-3	0	0	0	-9	-4	0
1	6	1	-4	-4	0	4	0	0	0	0	0	3	6	-2	-8
2	3	-2	-2	5	0	-2	0	1	-2	0	0	-3	-6	0	0
1	4	1	0	-2	0	0	0	1	-2	0	0	0	0	-2	-8
0	6	4	-3	-4	0	1	0	1	-2	0	0	3	6	-4	-16
0	9	0	-5	-3	1	1	1	1/2	-1/2	0	0	9/2	9/2	-4	-8
-2	13	2	-6	-5	0	2	0	1	-2	0	0	9	18	-8	-32
6	-3	-2	0	1	0	-2	4	-1/4	1/4	3/4	-3	3/4	3/4	-5/4	-1
6	-2	-2	-3	-2	2	7	-2	0	0	3/2	-6	-3	-3	5/2	2
6	-3	-2	-2	1	4	0	0	-1/4	1/4	9/4	-9	-9/4	-9/4	5/4	1
7	-5	-5	-1	7	2	1	-2	0	-1	-6	24	3	3	-4	0

$r_1$	$r_2$	$r_3$	$r_4$	$r_6$	$r_8$	$r_{12}$	$r_{24}$	$b_1$	$b_2$	$b_3$	$b_4$	$b_5$	$b_6$	$b_7$	$b_8$
6	-4	-2	3	0	-2	1	2	0	0	-3/2	6	0	3	3/2	-2
6	-3	-2	0	-3	0	10	-4	0	-1	-3	12	0	-3	-3	4
6	-4	-2	1	0	2	3	-2	0	-1	-9/2	18	0	0	-3/2	2
7	-7	-5	5	9	-2	-5	2	-1	1	6	-24	0	-3	-4	0
6	-6	-2	7	2	-2	-3	2	-1	1	9/2	-18	0	0	-5/2	-2
6	-7	-2	10	1	-4	0	0	0	-1	-9	36	0	9	3	-4
6	-4	-2	0	4	1	-2	1	-1/2	1/2	3	-12	-3/2	-3/2	0	0
6	-5	-2	3	3	-1	1	-1	0	-1	-6	24	0	3	0	0
5	-2	1	-2	-2	2	4	-2	0	-1	-3	12	-3	-3	1	4
5	-4	1	4	0	-2	-2	2	-1	1	3	-12	0	3	-1	-4
4	1	0	-1	-1	-1	-1	3	-1/2	1/2	0	0	3/2	9/2	-1	-4
4	2	0	-4	-4	1	8	-3	0	-1	0	0	-6	-9	2	8
4	1	0	-3	-1	3	1	-1	-1/2	-1/2	0	0	-9/2	-9/2	1	4
4	-2	0	6	0	-3	-2	1	-2	1	0	0	0	9	-2	-8
3	5	-1	-5	-3	-2	5	-2	0	-1	0	0	-9	-9	6	-8
3	3	-1	1	-1	-2	-1	2	-1	1	0	0	0	9	2	-8
8	-5	-4	1	5	-1	-3	3	1/2	-1/2	-3	12	-3/2	3/2	4	0
8	-4	-4	-2	2	1	6	-3	0	-1	-6	24	6	3	-8	0
8	-5	-4	-1	5	3	-1	-1	1/2	-3/2	-9	36	9/2	9/2	-4	0
8	-8	-4	8	6	-3	-4	1	2	-3	-18	72	0	9	8	0
7	-3	-1	-1	-1	0	3	0	0	0	3	-12	-3	-6	1	4
8	-6	-4	1	8	0	-3	0	1	-2	-12	48	3	6	0	0
7	-5	-1	3	1	0	-1	0	1	-2	-9	36	0	0	1	4
6	-3	2	0	-1	0	0	0	1	-2	-6	24	-3	-6	2	8
6	0	-2	-2	0	1	0	1	1/2	-1/2	0	0	-9/2	-9/2	5	4
6	-1	-2	1	-1	-1	3	-1	0	-1	0	0	0	-9	-6	8
4	4	0	-3	-2	0	1	0	1	-2	0	0	-9	-18	4	16
9	-4	-3	-2	0	2	4	-2	0	-1	-9	36	9	9	-9	-4
9	-6	-3	4	2	-2	-2	2	-1	1	9	-36	0	-9	-7	4
10	-5	-2	0	1	0	0	0	1	-2	-18	72	9	18	-2	-8

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Centre for Research in Algebra and Number Theory  
 School of Mathematics and Statistics  
 Carleton University  
 Ottawa, Ontario, K1S 5B6, Canada

AyseAlaca@cunet.carleton.ca  
 SabanAlaca@cunet.carleton.ca  
 ZaferAygin@cmail.carleton.ca